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Experiments on a non-smoothly-forced oscillator

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HIGHLIGHTS

- Experimental realization of previous theoretical study.
- Non-smooth oscillator with energy input at impact.
- Spectrograms showing alternative frequency-based bifurcation diagram.
- Excellent agreement between experimental data and numerical simulation.

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ABSTRACT

This paper describes some typical behavior encountered in the response of a harmonically-excited mechanical system in which a severe nonlinearity occurs due to an impact. Although such systems have received considerable recent attention (most of it from a theoretical viewpoint), the system scrutinized in this paper also involves a discrete input of energy at the impact condition. That is, it is kicked when contact is made. One of the motivations for this work is related to a classic pinball machine in which a ball striking a bumper experiences a sudden impulse, introducing additional unpredictability to the motion of the ball. A one-dimensional analog of a pinball machine was the subject of a detailed mathematical study in Pring and Budd (2011), and the current paper details behavior obtained from a mechanical experiment and describes dynamics not observed in a conventional (passive) impact oscillator.

1. Introduction

It is well established that non-smooth dynamical systems are capable of exhibiting a wide variety of fascinating behavior [1,2]. Some of this behavior, e.g., low-velocity grazing bifurcations, are typically *not* exhibited by smooth dynamical systems [3], and some experimental studies have been conducted [4–6]. The relative maturity of this subject (at least from a theoretical perspective) is reflected in a research monograph and the extensive list of references contained therein [7].

A recent paper [8] described some interesting behavior in a uni-directional analog of a pinball machine. That is, an otherwise linear oscillator experiences a sudden kick when it comes into contact with a barrier located at a given position. In their work, they showed some interesting bifurcation structure associated with this form of non-smooth forcing. The current paper describes an experimental study of just this type of system. Certain similarities and differences are noted. For example, in [8], they chose parameter values (for example, very high viscous damping) such that the system could be modeled as a first-order dynamical system, and thus facilitate mathematical modeling of the system as a discrete map. In the current study, the damping is relatively light (typical for most structural and mechanical systems) and the experimental data is compared with results from numerical simulation of the governing differential equation. Even in the absence of external periodic excitation the system is capable of exhibiting steady oscillatory behavior when the discrete addition of energy at impact balances the energy dissipation due to damping.

It is also worth mentioning that aspects of the unpredictability associated with the movement of a pinball is not unrelated to the roulette wheel for example [9,10].

2. The underlying linear system

2.1. Free vibration

Since the underlying, i.e., non-contacting, system is linear we can initially make use of some closed-form analytic solutions. It is the initiation of contact that causes the system to suddenly transition into nonlinear behavior.

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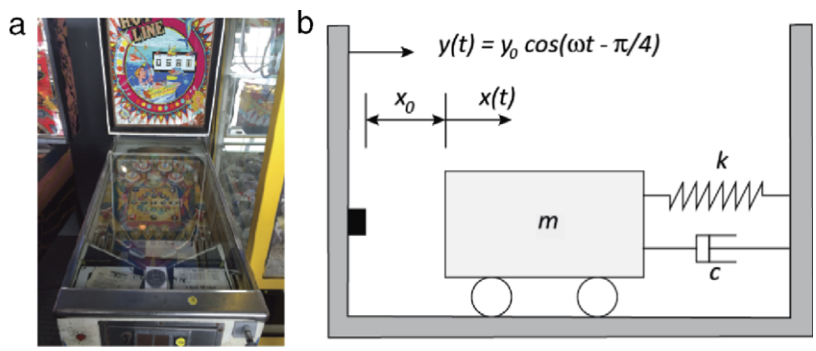

Fig. 1. (a) A pinball machine. (b) A spring–mass–damper with an immovable impact barrier and base excitation.

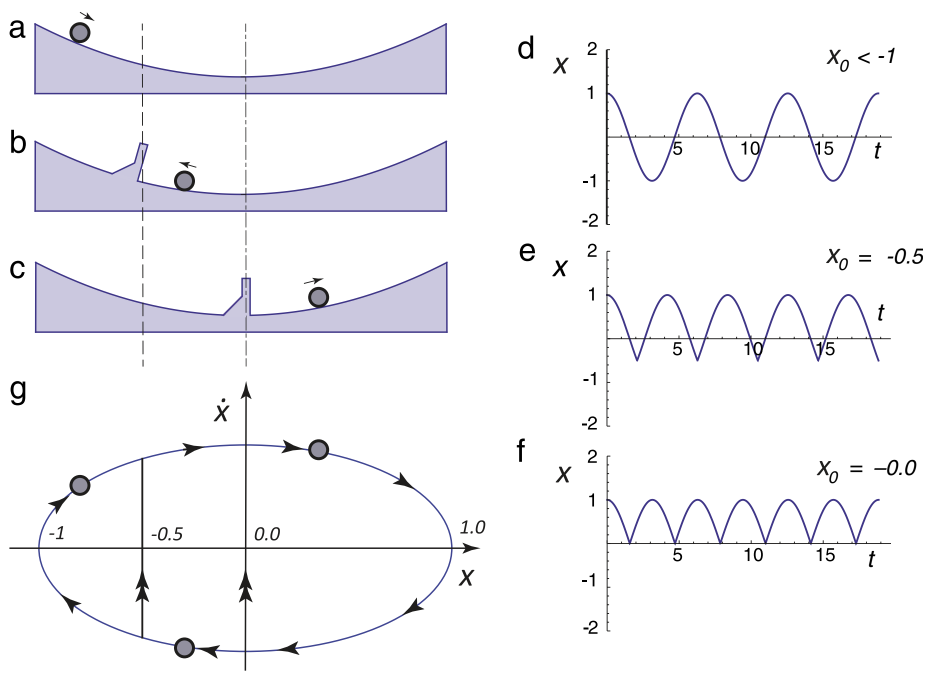

Fig. 2. The free response of an undamped impact oscillator. (a)–(c) A schematic of a small ball rolling on a parabolic surface, (d)–(f) some typical time series, and (g) elliptical phase trajectories cropped by the impact barrier.

Consider an old fashioned pinball machine as shown in Fig. 1(a). We choose to model this system using the one-dimensional system shown schematically in Fig. 1(b). Clearly, this is a somewhat simplistic analog but we shall see that it is still capable of exhibiting interesting dynamical behavior. This is the classic linear oscillator in which a point mass is attached to a linear (Hookean) spring and linear (viscous) dashpot. In the current study we consider the spring to provide the restoring force that replaces the effect of gravity and the pinball that rolls (downhill) due to the sloping surface.

Initially suppose we simply have $c = y(t) = 0$ and that there is no impact barrier. The equation of motion is simply $\ddot{x} = -\omega_n^2 x$, where $\omega_n = \sqrt{k/m}$ is a constant (the natural frequency), and given some non-zero initial conditions we get simple harmonic motion about the equilibrium position $x(t) = A\cos(\omega_n t + \phi)$. The velocity is $\dot{x}(t) = -A\omega_n \sin(\omega_n t + \phi)$ and it is useful to plot the phase trajectory $(x(t), \dot{x}(t))$ to show ellipses $(x^2 + (\dot{x}/\omega_n)^2 = A^2)$ and the familiar exchange of energy between potential and kinetic. This behavior is shown schematically in Fig. 2 in parts (a), (d) and (g). In part (a) we make use of the conceptual aid of a small ball rolling on a parabolic surface (which was also used directly as an experimental system in [5]). Part (d) shows a typical time series (with $x(0) = 1.0, \dot{x}(t) = 0$, and thus $A = 1$ and $\phi = 0$), with the corresponding elliptical phase trajectory in part (g).

Suppose we place an impact barrier to the left of the equilibrium position (at the bottom of the curve) and the small ball that can otherwise roll along the surface without friction. Now assume the ball is started from rest towards the right hand end, i.e., the initial conditions are $(x(0) = 1, \dot{x}(0) = 0)$ and the impact barrier prevents the ball from passing to the left of the barrier. If we further assume that the mass experiences a perfect elastic rebound then we can obtain the behavior shown in parts (b) and (e) when the barrier is located at $x_0 = -0.5$ and parts (c) and (f) when the barrier is located at $x_0 = 0.0$. Both of these are also contained within the ellipse in part (g) in which the mass experiences a sudden reversal of the sign of the velocity at impact. If we add linear viscous damping or allow some discrete loss of energy at impact (via a coefficient of restitution less than one) then the motion would ultimately decay.

2.2. Forced vibration

Now consider the case in which the support for the system is shaken harmonically. The equation of motion is

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 2\zeta\omega_n\dot{y} + \omega_n^2 y, \quad (1)$$

in which linear viscous damping has been added ($\zeta = c/c_c$), where $c_c = 2m\omega_n$. In general we will consider a lightly damped