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Large-scale weakly nonlinear perturbations of convective magnetic dynamos in a rotating layer

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HIGHLIGHTS

- Large-scale perturbations of convective dynamos are considered.
- Amplitude equations for the evolution of large-scale perturbations are derived.
- Numerical analysis of amplitude equations is performed.
- New mechanism for generation of large-scale magnetic field by convection is found.

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ABSTRACT

We present a new mechanism for generation of large-scale magnetic field by thermal convection which does not involve the α-effect. We consider weakly nonlinear perturbations of space-periodic steady convective magnetic dynamos in a rotating layer of incompressible electrically conducting fluid that were identified in our previous work. The perturbations have a spatial scale in the horizontal direction that is much larger than the period of the perturbed convective magnetohydrodynamic state. Following the formalism of the multiscale stability theory, we have derived the system of amplitude equations governing the evolution of the leading terms in the expansion of the perturbations in power series in the scale ratio. This asymptotic analysis is more involved than in the cases considered earlier, because the kernel of the operator of linearisation has zero-mean neutral modes whose origin lies in the spatial invariance of the perturbed regime, the operator reduced on the generalised kernel has two Jordan normal form blocks of size two, and simplifying symmetries of the perturbed state are now missing. Numerical results for the amplitude equations show that a large-scale perturbation, periodic in slow horizontal variable, either converges to a small-scale neutral stability mode with amplitudes tending to constant values, or it blows up at a finite slow time.

1. Introduction

A fundamental problem in astrophysics is to understand the sources of magnetic fields that are featured by many astrophysical bodies such as the Sun and the Earth. It is generally accepted that the magnetic fields are generated by hydromagnetic processes in the melted or fluid-like interiors of the bodies. This idea goes back to J. Larmor [1,2]. It is widely believed that the so-called α-effect plays a prominent role in such processes. Such a mechanism of magnetic field generation was first suggested by E. Parker [3,4]. It relies on the frozenness of magnetic field into the conducting medium, when magnetic diffusion is negligible, implying that a small eddy in turbulent flow deforms the magnetic field force line into a loop. If this effect does not disappear when averaged over many loops, it gives rise to a mean electromotive force (e.m.f.), which can be parallel to the mean unperturbed magnetic field and can amplify the original mean field.

The theory of mean-field electrodynamics (MFE) is developed around a similar central idea [5,6]: Suppose the flow and magnetic field are split into the mean and fluctuating parts, and the interaction of the fluctuating parts of the flow and the field yields a non-zero mean

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e.m.f. The MFE theory postulates that, at least in the case of magnetohydrodynamic (MHD) turbulence, the latter is related to the mean magnetic field (the coefficient of proportionality is traditionally denoted by α, and, accordingly, the phenomenon is called the "α-effect") combined, in some MFE models, with the spatial gradient of the mean field.

The MFE theory does not fully justify such relations. An insight into their mathematical roots is provided by the multiscale stability theory (MST). It treats an idealised case, in which turbulence is modelled by a small-scale laminar flow, periodic in space and time (see [7]). In contrast with MFE, mathematically rigorous results are then obtained by applying the asymptotic theory of PDE homogenisation to the problem of linear or weakly nonlinear stability of small-scale states. Purely hydrodynamic [8,9] perturbations, the kinematic dynamo problem [10–12] (which is an instance of the general linear MHD stability problem, where the perturbed state is amagnetic, the flow and magnetic components of the perturbation therefore decouple, and one focuses on the magnetic perturbation), and full perturbations of forced MHD or convective MHD states [13–16] (see also [7, Chap. 6–9]) were considered. Perturbations are supposed to involve spatial and temporal scales that are much larger than the respective periods of the perturbed states. The perturbations are linear combinations of amplitude-modulated small-scale modes (i.e., eigenfunctions of the linearisation whose periods coincide with those of the perturbed states) associated with the same eigenvalue. The coefficients depend exclusively on slow temporal and spatial variables. They are usually called amplitudes, and the equations governing their dynamics are called amplitude equations. The evolution of such large-scale perturbations is essentially controlled by the associated eigenvalue of the constituting small-scale modes—only neutral (belonging to the kernel of linearisation) small-scale modes can instigate instability in the presence of large scales; consequently, MST usually focuses on large-scale amplitude modulation of neutral modes. Since typically small-scale neutral modes have non-zero means, amplitude equations for large-scale perturbations of forced MHD states (convective or not) are mean-field equations similar to those considered in MFE theory. However, as we will see, this is not always the case: translation-invariant physical systems, such as free thermal hydromagnetic convection in a horizontal layer investigated in the present paper, possess zero-mean neutral small-scale stability modes. Consequently, the mean-field description of the dynamics of large-scale perturbations of such physical systems is inadequate.

We intend to carry out a detailed investigation of stability to large-scale weakly nonlinear perturbations of the steady and time-periodic regimes of magnetic field generation by free thermal convection of rotating electrically conducting fluid in a horizontal layer, that were determined numerically in [17,18]. Here we consider space-periodic convective MHD steady states constituting the branch $\mathrm{S}_8^{\mathrm{R}1}$ [17]. The group of symmetries of this steady state is generated by the symmetry about the vertical axis, x_3, (note that this is not axisymmetry, see Section 2) [17,7] and the superposition of reflection about the midplane with translation by half a period in the horizontal direction x_1. This group of symmetries is smaller than other ones, typical for the parameter values considered *ibid.*, and the system of amplitude equations, derived in [7], is inapplicable for large-scale perturbations of states having this group. The symmetry about a vertical axis implies that the steady state does not possess the α-effect. The large-scale dynamics is due to the interplay of the combined eddy diffusivity and eddy advection. We find that their interaction cannot sustain stationary generation of large-scale magnetic field: a large-scale perturbation from the class which is described by the multiscale formalism either converges to a small-scale neutral stability mode, or it blows up at a finite time.

The paper is organised as follows. In Section 2 we derive the system of amplitude equations for steady states of free hydromagnetic convection, which have the same group of symmetries as those comprising the branch $\mathrm{S}_8^{\mathrm{R}1}$. In Section 3 we present results of numerical analysis of the system of amplitude equations for several states belonging to this branch. Finally, we make remarks triggered by our investigation.

2. The multiscale formalism for large-scale perturbations of free hydromagnetic convection

In this section we derive amplitude equations for large-scale perturbations of small-scale steady free hydromagnetic convection in a horizontal layer of electrically conducting fluid rotating about the vertical axis. A field, depending only on the fast spatial variables, $\mathbf{x}$, and time, t, is called small-scale; if, in addition, the field depends on the slow horizontal spatial variables, $\mathbf{X} = \varepsilon(x_1, x_2)$ and on the slow time, $T = \varepsilon^s t$, where $s \geq 1$, it is called large-scale. We will use asymptotic methods that are standard within the MST approach. We assume that the perturbed state is symmetric about the vertical axis and has equal periods $\mathscr{P}$ in x_1 and x_2; further assumptions are introduced where they become relevant.

A three-dimensional field $\mathbf{f}$ is called symmetric about a vertical axis passing through point $(a_1, a_2, 0)$ when the following conditions are satisfied [17,7]:

$$f_1(a_1 - x_1, a_2 - x_2, x_3) = -f_1(a_1 + x_1, a_2 + x_2, x_3),$$
$$f_2(a_1 - x_1, a_2 - x_2, x_3) = -f_2(a_1 + x_1, a_2 + x_2, x_3),$$
$$f_3(a_1 - x_1, a_2 - x_2, x_3) = f_3(a_1 + x_1, a_2 + x_2, x_3).$$

In particular, this symmetry implies that the flow is vertical everywhere on the axis. Together with the $\mathscr{P}$-periodicity in the horizontal directions, the symmetry about the Cartesian axis x_3 implies that the field is also symmetric about vertical axes through points $\mathscr{P}\mathbf{n}/2$, where $\mathbf{n} = (n_1, n_2, 0)$ has integer components.

2.1. Small-scale convective hydromagnetic steady states and their perturbations

The state, whose stability we examine, is governed by the Navier–Stokes, magnetic induction and heat transfer equations. In the coordinate system, co-rotating with the fluid layer about the axis x_3, they are [19]

$$\partial\mathbf{V}/\partial t = \nu\nabla^2\mathbf{V} + \mathbf{V} \times (\nabla \times \mathbf{V}) - \mathbf{H} \times (\nabla \times \mathbf{H}) + \beta\Theta\mathbf{e}_3 + \tau\mathbf{V} \times \mathbf{e}_3 - \nabla P, \quad (1)$$

$$\partial\mathbf{H}/\partial t = \eta\nabla^2\mathbf{H} + \nabla \times (\mathbf{V} \times \mathbf{H}), \quad (2)$$

$$\partial\Theta/\partial t = \kappa\nabla^2\Theta - (\mathbf{V} \cdot \nabla)\Theta + \delta V_3, \quad (3)$$

$$\nabla \cdot \mathbf{V} = \nabla \cdot \mathbf{H} = 0. \quad (4)$$