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Adaptive bipartite consensus on coopetition networks

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HIGHLIGHTS

- Adaptive bipartite consensus problem is formulated for coopetition networks.
- The second-order agent dynamics suffers from unknown time-varying disturbances.
- A bridgebuilder agent is introduced to intervene the final consensus state.
- Adaptive bipartite consensus strategy is designed for each agent.
- Sufficient conditions are given for consensus convergence of the multi-agent system.

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ABSTRACT

In this paper, a bipartite consensus tracking problem is considered for a group of autonomous agents on a coopetition network, on which the agents interact cooperatively and competitively simultaneously. The coopetition network involves positive and negative edges and is conveniently modeled by a signed graph. Additionally, the dynamics of all the agents are subjected to unknown disturbances, which are represented by linearly parameterized models. An adaptive estimation scheme is designed for each agent by virtue of the relative position measurements and the relative velocity measurements from its neighbors. Then a consensus tracking law is proposed for a new distributed system, which uses the relative measurements as the new state variables. The convergence of the consensus tracking error and the parameter estimation are analyzed even when the coopetition network is time-varying and no more global information about the bounds of the unknown disturbances is available to all the agents. Finally, some simulation results are provided to demonstrate the formation of the bipartite consensus on the coopetition network.

1. Introduction

Recent years have witnessed that a surge of attention has been paid to study the consensus emergence of multi-agent systems where agents interact in the neighborhood according to some simple local rules. When the interaction relationship between agents is cooperative or competitive, diverse collective behaviors, such as consensus, polarization, or fragmentation can emerge in a macroscopical fashion from the microcosmical interactions among agents [1]. Cooperation is a common relationship between agents in a social system or a networked system. The emergence of a global consensus is widely investigated for cooperative systems, whose interaction network is normally modeled by a graph with nodes representing agents and (positive) edges describing their pairwise collaboration (see [2] and references therein). However, competition is another inherent relationship between agents. For example, the antagonistic relationship is common in human systems [3–5] and ecological systems [6]. In fact, competition and cooperation normally coexist in natural and engineering systems.

In many real world scenarios, another type of "consensus" phenomenon has been observed for a long time, where all the agents reach a final state with identical magnitude but opposite sign. Hereafter, we call such kind of collective behavior as *bipartite consensus* or *anti-synchronization*. For example, a polarization often happens in a two-coalition community such that opposite opinions are held by two fractions [3,4]. Anti-synchronization phenomena have also been observed in synchronization between two pendulum clocks, salt-water oscillators experiments and chaotic systems [7,8]. In the field of Statistics Physics, there is a classical model to describe the crystal magnetization phenomenon—Ising model, where the agents (electrons) interact each other through ferromagnetic or antiferromagnetic interactions and anti-synchronization emerge under some critical condition [9]. In order to study bipartite consensus, the interaction networks among agents are generally modeled by signed graphs with positive/

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negative edges [10]. In the field of Strategic Management, Brandenburger and Nalebuff coined the term "coopetition" to describe the coexistence of competition and cooperation [11]. In order to analyze the emergence of bipartite consensus on coopetition networks, people heavily rely on structural balance, which is an important property in the signed graph theory. A coopetition network with structural balance can be partitioned into two subnetworks such that each subnetwork contains only positive edges while all edges joining different subnetworks are negative [10,12]. In the community of control theory, the evolution of a first-order collective dynamics is analyzed by using the notion of structural balance [13–16]. A sufficient and necessary condition was presented in [14] to ensure that bipartite consensus can be reached if and only if the signed graphs associated with multi-agent systems are strongly connected and structurally balanced. Hu and Zheng investigated sufficient conditions for consensus, polarization and fragmentation for multi-agent systems if the associated coopetition networks are heterogeneous or homogeneous, and structurally balanced, structurally unbalanced, or vacuously balanced in [16]. As an extension of the results reported in [14], a bipartite consensus problem was considered in [17] for a high-order multi-agent system described by a linear time-invariant system.

In this paper, a bipartite consensus problem is investigated for a group of autonomous agents with second-order dynamics suffering from unknown time-varying disturbances. All the agents interact cooperatively and competitively simultaneously. The agent dynamics suffers from unknown disturbances. Moreover, there exists an external agent, who has a self-active dynamics and plays an intervention role in the multi-agent system. The aim of this paper is to design a consensus tracking law for each agent and provide some sufficient conditions to realize a bipartite consensus on the state of the external agent. Thus, it is crucial to develop an adaptive disturbance rejection control for the multi-agent system. In cooperative tracking control problems, when there exist unknown disturbances in the agent dynamics, decentralized adaptive estimation designs were proposed to reconstruct a prescribed reference velocity and to track the reference velocity by using both relative position and velocity measurements in [18]. An adaptive synchronization control was designed for a first-order nonlinear leader–follower system having unknown dynamics in [19]. When the agent dynamics is subjected to bounded unknown disturbances and only the position information of the leader can be measured, in [20], a distributed "observer" was firstly designed to estimate the velocity of the leader and then a tracking control was given for each follower by using the relative position measurement and the velocity estimate. However, it was assumed therein that the input of the leader is a common policy known by all followers and at the same time, the velocity of each follower can be measured. In order to relax these constraints, an adaptive tracking control was presented to solve a second-order leader-following problem by using only relative position measurements in [21]. However, few results have been found for consensus tracking on coopetition networks. Thus the contribution of this paper is twofold. One is to build a second-order collective dynamics with unknown time-varying disturbances on coopetition networks. The other is to design decentralized adaptive laws to estimate unknown disturbances and consensus tracking laws to realize the bipartite consensus tracking on coopetition networks.

The remainder of this paper is organized as follows. In Section 2, coopetition networks are modeled by signed graphs and some notations about structural balance are presented. At the same time, a collective dynamics is modeled by a second-order multi-agent system with unknown disturbances. In Section 3, both an adaptive estimation scheme and a consensus tracking law are proposed for agents by using relative measurements from neighbors. Meanwhile, the consensus tracking convergence is analyzed for time-varying coopetition networks. Furthermore, if a persistent excitation condition is assumed for the unknown disturbances, the convergence of the parameter estimation is also guaranteed. In Section 4, some simulation results are provided to demonstrate the formation of the bipartite consensus tracking on the coopetition network. Finally, some concluding remarks are given in Section 5.

2. Problem formulation

2.1. Coopetition network modeling

When we regard an agent as a node and the interactions between two agents as positive/negative edges, it is helpful to use a signed graph to describe a coopetition network. The positive and negative edges in a signed graph represent, respectively, the cooperative and competitive interactions in the coopetition network. Herein, we give an intuitive illustration of coopetition networks in Fig. 1. In Fig. 1(a) and (b), the two networks $\mathcal{G}^s$ and $\mathcal{G}^s_{Tree}$ are signed graphs, which have positive and negative edges. The positive and negative edges are denoted by blue solid and red dash lines, respectively. The network $\mathcal{G}^s$ has two antagonistic subnetworks $\mathcal{V}_1$ and $\mathcal{V}_2$. The edges are positive within each subnetwork and are negative between the two subnetworks. The graph $\mathcal{G}^s_{Tree}$ is a spanning tree of $\mathcal{G}^s$ and the node 1 is a root node.

Formally, a **signed graph** is a graph $\mathcal{G}^s = \{\mathcal{V}, \mathcal{E}, A\}$, where $\mathcal{V} = \{1, \ldots, N\}$ is a set of nodes, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of edges, and A is an adjacency matrix describing the edge information of a positive or negative sign. The nonzero element a_{ij} of A is attached to the edge $(i, j) \in \mathcal{E}$, i.e., $a_{ij} \neq 0$ if and only if $(i, j) \in \mathcal{E}$. In this way, agent i and agent j can communicate information to each other. The edge set $\mathcal{E} = \mathcal{E}^+ \bigcup \mathcal{E}^-$, where $\mathcal{E}^+ = \{(j, i)|a_{ij} > 0\}$ and $\mathcal{E}^- = \{(j, i)|a_{ij} < 0\}$ are the sets of positive and negative edges, respectively. If all edges are positive and $\mathcal{E}^- = \varnothing$, the signed graph is reduced to an unsigned graph or graph $\mathcal{G}$. A **path** is a sequence of edges of the form $(i_1, i_2), (i_2, i_3), \ldots, (i_{l-1}, i_l)$ with distinct nodes with length $l - 1$. A **cycle** is a path beginning and ending with the same nodes. A signed graph $\mathcal{G}^s$ is said to be connected if there is a path between any pair of distinct nodes. A **tree** is a connected graph without cycles. A **spanning tree** of a graph is a tree that includes all of the nodes and some of the edges of the graph. In Fig. 1(a), the nodes 1, 7, 8, 9 and 2 form a cycle. In Fig. 1(b), $\mathcal{G}^s_{Tree}$ is a spanning tree of $\mathcal{G}^s$.

A cycle in coopetition networks generally contains positive and negative edges. If the product of the weights a_{ij} in a cycle is positive, then we say that the cycle is positive; and negative, otherwise. Obviously, the cycle $1 \to 7 \to 8 \to 9 \to 2 \to 1$ is positive since there are two negative edges $1 \leftrightarrow 7$ and $2 \to 9$ in $\mathcal{G}^s$ in Fig. 1(a). A coopetition network $\mathcal{G}^s$ is said **structurally balanced** if all of its cycles are positive [10], and $\mathcal{G}^s$ is said **structurally unbalanced** if one of its cycles is negative. It is noticed that the existence of cycles is a necessary condition of structural balance. If a network $\mathcal{G}^s$ has no cycles, it is said **vacuously balanced** [12]. For example, the coopetition network $\mathcal{G}^s$ is structurally balanced while $\mathcal{G}^s_{Tree}$ is vacuously balanced in Fig. 1. A coopetition network is said to be **homogeneous** if all the interactions are cooperative or competitive, and **heterogeneous**, otherwise. Thus homogeneous coopetition networks have two classes: all-positive networks (i.e., all of the edges are positive) and all-negative networks (i.e., all of the edges are negative).

Notice that there is a common phenomenon that a structurally balanced coopetition network consists generally of two antagonistic subnetworks. The interactions are cooperative within each subnetwork, but competitive between the two subnetworks. Additionally, a coopetition network is called bipartite if it can be partitioned into two subnetworks such that all the interactions only