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Two-dimensional expansion of a condensed dense Bose gas

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HIGHLIGHTS

- We study the expansion of strongly interacting Bose gas.
- Flow past obstacles generates shock waves.
- Shock waves of weak and strong interacting Bose gas showed to be similar.

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ABSTRACT

We study the expansion dynamics of a condensate in a strongly interacting Bose gas in the presence of an obstacle. Our focus is on the generation of shock waves after the Bose gas has passed the obstacle. The strongly interacting Bose gas is described in the slave-boson representation. A saddle-point approximation provides a nonlinear equation of motion for the macroscopic wave function, analogous to the Gross–Pitaevskii equation of a weakly interacting Bose gas but with different nonlinearity. We compare the results with the Gross–Pitaevskii dynamics of a weakly interacting Bose gas and find a similar behavior with a slower behavior of the strongly interacting system.

1. Introduction

Many interesting features of a trapped Bose gas are described by the Gross–Pitaevskii (GP) equation. This includes dynamical studies of the Bose–Einstein condensate (BEC), such as the expansion of a BEC after switching off the trapping potential or the interference of two separate BEC's, has been performed with remarkable accuracy [1]. The GP equation provides the macroscopic quantum state of the condensed part of the Bose gas. It can be derived from a microscopic statistical model of many-body states at temperature T in the limit $T = 0$ by a saddle-point approximation. However, the GP equation is restricted to a dilute Bose gas, with less then one particle in the scattering volume. More recent experiments with optical lattices have revealed that a much richer physics appears in a dense (or strongly interacting) Bose gas [2]. The idea is that a static periodic potential of the optical lattice is provided by counter-propagating Laser fields, where particles occupy the local minima of the periodic potential. As soon as there is more than one particle per minimum the Bose gas must be considered as strongly interacting. An immediate effect of stronger interaction is the depletion of the condensate caused by the collision of particles. This is also known from the famous example of interacting bosons in form of superfluid helium. If the density of the Bose gas increases further we can even destroy the condensate completely and create a new quantum state in the form of a Mott insulator. In contrast to a BEC, the Mott insulator is characterized by local conservation of the particle number (i.e. $n = 1, 2, \ldots$ particles per minimum of the optical lattice) but without phase coherence. In a trapped Bose gas, contrary to a translational invariant Bose gas, both states can co-exist in the same system, which is known as the wedding-cake structure: at commensurate densities the system is in a Mott state with constant density and at incommensurate densities the system is in a Bose–Einstein condensed state with spatially changing density.

These strongly interacting systems cannot be described within the GP approach because the latter only takes into account the condensate and neglects the interaction with non-condensed particles. In particular, depletion of the condensate or the formation of a Mott insulating state is not accessible by the GP approach. This problem has been addressed in a number of different approaches [3,4]. A very direct approach is an extension of the GP equation that is able to take into account the interaction with the

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non-condensed particles. It is known as the slave-boson (SB) approach and provides also a nonlinear Schrödinger equation for the macroscopic quantum state of the condensed part of the Bose gas. The interaction with the non-condensed particles leads to a modified nonlinearity though. For a dilute Bose gas the nonlinearity is the same as that of the GP equation but in the regime of the dense Bose gas it is weaker than that of the GP equation. The equation of the strongly interacting Bose gas also indicates the disappearance of the condensate when we approach the Mott-insulating state at higher density. Previous studies of a dense trapped Bose gas have shown that the BEC can be completely destroyed at the trapping center due to depletion at higher densities [5–8]. On the other hand, the vortex structure is not much affected in the strongly interacting system because the density of the condensate is low in the vicinity of the vortex core [9].

The two-dimensional expansion of a dilute BEC past an obstacle is a subject of intensive theoretical and experimental studies. A relevant parameter in these studies is the Mach number M. It is defined as the ratio of the asymptotic velocity of the flow to the sound velocity in the medium [10]. For subsonic Mach numbers in the range $0.5 \lesssim M \lesssim 0.9$, it was reported the generation of pairs of vortices and antivortices [11]. In case of a supersonic flow, the generation of oblique dark solitons inside, and Kelvin ship waves outside the Mach cone (imaginary lines drawn from the obstacle at angles $\pm\arcsin(1/M)$ with the horizontal axis) were found [12–16]. It was shown in [17] that these dark solitons are convectively unstable for large enough flow velocity ($M \gtrsim 1.5$), i.e., practically stable in the region around the obstacle. A general theory on dispersive shock waves for supersonic flow past an extended obstacle was developed in [18] and a review paper on dark solitons in BEC can be found at [19]. Experiments addressing this problem were described in [20–22]. Recently, a renewed theoretical interest in this issue was brought by the observation of an alternating vortex emission for a suitable set of parameters. These are analogous to the "von Kármán vortex street" in classical dissipative fluids [23].

In this paper we shall study the effect of the interaction between the BEC and the non-condensed particles in a dynamical situation, where a BEC is released from a parabolic trap and passes an obstacle. The obstacle is modeled by an impenetrable disk. Due to a complex interference the macroscopic wave function will experience strong density fluctuations. The results of our numerical simulation, based on the strongly interacting gas (SIG) equation, will be compared with previous calculations, based on the GP equation. The paper is organized as follows: We briefly introduce the SIG equation and compare it with GP equation. Then we present the results of our numerical simulation for an expanding cloud in two dimensions that passes an obstacle. Finally, we discuss these results and compare them with those of the GP approach.

2. Slave-boson approach

We start from a Bose gas with hard-core interaction of a given radius a, representing an effective scattering length. Then the Bose gas can be approximated by a lattice gas with lattice constant a. In other words, a provides the shortest relevant length scale in our Bose gas. This scale remains in the Bose gas even after its release from the trap. In general, a strongly interacting Bose gas has two constituents, namely condensed particles and non-condensed particles. This is the case even at zero temperature. The interplay of all these particles can be described by the slave-boson approach [5–9]. Although this is a many-body picture, the macroscopic wave function of the condensate is extracted by a variational procedure with respect to the density of the condensate analogous to the derivation of the GP equation from the weakly interacting many-body Bose gas model. The corresponding effective Hamiltonian of the macroscopic condensate state $\Phi(\mathbf{r})$ is

$$H_\Phi = -\frac{Ja^2}{6}\,\Delta + J\,(\alpha_1 - \alpha_2\chi)\,, \tag{1}$$

where Δ is the three-dimensional Laplacian. Moreover, χ is a term that is nonlinear in $|\Phi|^2$:

$$\begin{aligned}\chi(\mathbf{r}) &= \frac{\partial \log Z(\mathbf{r})}{\partial|\Phi(\mathbf{r})|^2}\\ &= \frac{1}{2}\frac{1}{Z(\mathbf{r})}\int_{-\infty}^{\infty} e^{-\varphi^2}\left(\frac{\cosh\gamma(\mathbf{r})}{\gamma(\mathbf{r})^2} - \frac{\sinh\gamma(\mathbf{r})}{\gamma(\mathbf{r})^3}\right)d\varphi,\end{aligned} \tag{2}$$

where $Z(\mathbf{r})$ is the integral expression

$$Z(\mathbf{r}) = \int_{-\infty}^{\infty} e^{-\varphi^2}\,\frac{\sinh\gamma(\mathbf{r})}{\gamma(\mathbf{r})}d\varphi \tag{3}$$

with

$$\gamma(\mathbf{r}) = \sqrt{\left(\varphi + \mu/2\right)^2 + |\Phi(\mathbf{r})|^2}. \tag{4}$$

The coefficients are $\alpha_1 = 1+\alpha$, $\alpha_2 = \alpha(1+1/\alpha^2)$, where α is a numerical constant $\alpha \approx 1/5.5$ [5,9]. The kinetic energy parameter J is associated with the mass of the bosons m by the relation

$$\frac{Ja^2}{6} = \frac{\hbar^2}{2m}.$$

All model parameters are measured in terms of the length scale a and the energy scale J. The dimensionless parameter μ is a one-particle chemical potential that is associated with the density of bosons. This can be understood as a potential that controls the exchange of particles with the gas outside the trapped cloud by assuming that the cloud is a grand-canonical ensemble of atoms. Then the value of μ fixes the number of bosonic atoms in equilibrium.

The results of the SB approach can be compared with those of the Bogoliubov approach in the regime of a dilute condensate. The quasiparticle spectrum of a homogeneous condensate with condensate density n_0 reads [8]

$$E_{\mathbf{k}} = \sqrt{\epsilon_{\mathbf{k}}\,(2\,gn_0 + \epsilon_{\mathbf{k}})}. \tag{5}$$

The two approaches are distinguished by the parameter g, which is the interaction constant of the weakly interacting Bose gas g in the Bogoliubov approach and a renormalized effective interaction in the case of the SB approach. In the latter g is a function of the chemical potential μ and the temperature with a maximum at $\mu = 0$ [7]. Thus, for $\epsilon_{\mathbf{k}} = \hbar^2k^2/2m$ the sound velocity reads $v_s = \sqrt{gn_0/m}$. Moreover, the healing length can also be extracted from the fluctuations of the SB approach as $\xi = \hbar/\sqrt{2mgn_0}$ [8], which also agrees with the result of the Bogoliubov approach. This means that the sound velocity and the healing length of the strongly interacting Bose gas are renormalized in comparison with the weakly interacting Bose gas. The behavior of the condensate density and the renormalized interaction parameters of the SB approach are depicted in Figs. 2, 3, respectively.

The obstacle is included by choosing specific boundary conditions for the macroscopic wave function. In our case this is a hard disk, where the wavefunction vanishes inside the disk. Finally, the number of condensed bosons N_0 is determined by an integral of $|\Phi(\mathbf{r})|^2$ over the entire volume of a three-dimensional Bose gas as

$$N_0 = \frac{1}{a^3}\frac{1}{(1+1/\alpha)^2}\int|\Phi(\mathbf{r})|^2 d^3r. \tag{6}$$