AN ANALYSIS OF SOLUTIONS OF THE ONE-DIMENSIONAL HUBBARD MODEL

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We discuss the one-dimensional Hubbard model of N sites half-filled chain, on the example of a ring with four nodes, in context of the diagonalization process. We consider translational and unitary symmetry of the problem, given by the C_N, and the direct product of two unitary groups SU(2) × SU(2), respectively. We construct nine of the sixteen eigenstates which are not dependent on parameters U and t using the irreducible basis of the appropriate groups, and study the seven remaining states using graphical representations for different ranges of the coefficients. We discuss the different and often conflicting behaviors of the kinetic and potential parts of the Hubbard Hamiltonian in different areas of a parameter space.

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1. Introduction

Hubbard model [1] is of considerable interest in the theory of strongly correlated electron systems. In spite of a substantial simplification of the model compared to the real physical situation of interacting electrons in a crystal, the obtained results may explain insulating, magnetic, and even superconducting effects in a solid. Hubbard model has been applied to the understanding of many sophisticated physical problems like ferromagnetism, antiferromagnetism, the Mott transition, high-temperature superconductivity or Bose–Einstein condensate in cold optical lattice [2–4], etc. Despite its apparently simple structure the exact solution exists only for one spatial dimension, given in 1968 year by Lieb and Wu [5], by using the method of Yang [6] and [7] from the year before, and is not easy to analyze. In general, there are several approximation techniques, among others mean field-theory, various Green's function decoupling schemes and functional methods. We continue the work with one spatial dimension, which is of much importance for its sake of their possible generalizations to higher dimensions, and as the exactly solvable model [8, 9]. In addition, the one-dimensional case may become more important due to its possible applications in intensively studied carbon nanotubes [10], one-dimensional organic superconductors [11], or one-dimensional organic ferromagnet [12]. There is also

possibility of modelling the one-dimensional Hubbard model of fermionic quantum gas loaded into an optical lattice [13], which is a promising candidate for quantum information processing.

2. The Hamiltonian

The Hilbert space $\mathcal{H}$ of N_e electrons, moving along the chain with N nodes, can be created starting from defining the *single-node* space h_j, with the basis consisting of vectors denoting all possible occupations of one node. Since we are dealing with fermions,

$$\dim h_j = 4, \qquad h_j = lc_{\mathbb{C}}\{\pm, \emptyset, +, -\}, \tag{1}$$

where $\emptyset$ denotes the empty node, $+$ and $-$ stand for one-node spin projection equal to $\frac{1}{2}$ and $-\frac{1}{2}$, respectively, $\pm$ denotes the double occupation of one node by two electrons with different spin projections, and $lc_{\mathbb{C}}A$ stands for the linear closure of a set A over the complex field $\mathbb{C}$. One can obtain the final space $\mathcal{H}$ of all quantum states of the system in the following way

$$\mathcal{H} = \prod_{j=1}^{N} \otimes\, h_j, \qquad \mathcal{H} = \sum_{N_e=0}^{2N} \oplus\, \mathcal{H}^{N_e}, \tag{2}$$

where $\mathcal{H}^{N_e}$ denotes the space with fixed number of electrons N_e.

The dynamics of a finite set of interacting electrons, occupying a one-dimensional chain consisting of N atoms, can be described by the Hubbard Hamiltonian in the form

$$\hat{H} = -t \sum_{i\in\tilde{2}} \sum_{j\in\tilde{N}} (\hat{a}^{\dagger}_{ji}\hat{a}_{j+1i} + \hat{a}^{\dagger}_{j+1i}\hat{a}_{ji}) + U \sum_{j\in\tilde{N}} \hat{n}_{j\,+}\hat{n}_{j\,-}, \tag{3}$$

where $\tilde{N} = \{j = 1, 2, \ldots, N\}$ denotes the set of atoms of the chain, $\tilde{2} = \{i = +, -\}$, $\hat{n}_{ji} = \hat{a}^{\dagger}_{ji}\hat{a}_{ji}$, and finally $\hat{a}^{\dagger}_{ji}$, $\hat{a}_{ji}$ are the canonical Fermi operators, that is the creation and anihilation operators of electron of spin i, on the site j. One can observe that electrons behave as *waves* in the first component of the Hamiltonian (3), while they behave as *particles* in the second with assumption of the occurrence of electron-electron interaction with the characteristic constant interaction denoted by U [14]. In general, U can have any value, with $U < 0$ and $U > 0$ ($U \gg 0$ [15, 16]) responsible for attraction and repulsion, respectively, $U = 0$ stands for no effect or a plain gas of fermions.

The set of all linearly independent vectors called *electron configurations* [17] provides the initial, orthonormal basis of the Hilbert space $\mathcal{H}$. These configurations are defined by the mapping

$$f : \tilde{N} \to \tilde{4}, \qquad \tilde{4} = \{\pm, \emptyset, +, -\}, \tag{4}$$

and constitute the N-sequences of the elements from the set $\tilde{4}$

$$|f\rangle = |f(1)f(2)\ldots f(N)\rangle = |i_1 i_2 \ldots i_N\rangle, \qquad i_j \in \tilde{4}, \qquad j \in \tilde{N}, \tag{5}$$