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Transitions between symmetric and nonsymmetric regimes in binary-mixture convection

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HIGHLIGHTS

- We study the dynamics of binary-mixture convection on a laterally heated cavity.
- The large range of used parameters connect results from separation ratio $S = 0$ to $S = -1$.
- Both steady and time-dependent solutions have been calculated.
- A complex scenario of connected bifurcations has been found.
- Identification of high codimension bifurcations.

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ABSTRACT

We present here a comprehensive picture of the different bifurcations found for small to moderate Rayleigh number in binary-mixture convection with lateral heating and negative separation ratio (S). The present work connects the symmetric regime found for pure fluid ($S = 0$) (Mercader et al., 2005) with the fundamentally nonsymmetric regime found for $S = -1$ (Meca et al., 2004) [2,3]. We give a global context as well as an interpretation for the different associations of bifurcations found, and in particular we interpret an association of codimension-two bifurcations in terms of a higher codimension bifurcation never found, to our knowledge, in the study of an extended system.

1. Introduction

The study of convecting systems has a long and rich history, and many of the techniques developed for their study have been later successfully applied to broader categories of problems, such of pattern-forming systems [4]. In particular, the study of the different instabilities has motivated some of the developments in the field of dynamical systems theory which in turn has led to a systematic way of approaching the study of convecting systems.

The study of convection in two-component systems, usually referred to as double-diffusive convection, was originally motivated by oceanographic problems [5], i.e. by the study of the relative role played by heat and solute (salt) diffusion in the development of convecting structures in the ocean. The application of double-diffusive convection to this and other problems, ranging from alloy solidification to stellar physics has received much attention in the past [6–8].

In this paper we study a binary-mixture, that is, a fluid with two miscible components in which thermodiffusion (Soret effect) is present. The influence of the Soret effect as a driving force that could build up solutal gradients was investigated in [9], and the phenomenology uncovered and experimentally confirmed showed that the system behaved similarly to double-diffusive systems. This coincidence was later explained [10] by noticing a correspondence between Soret convection and the double-diffusive case for certain boundary conditions. This correspondence does not apply in this work.

While the most common setup in convection experiments is that of a fluid layer heated from below, other configurations are possible. In many situations (technological, oceanographical, etc.) lateral thermal gradients arise. The case of lateral thermal and

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solutal gradients is the one that concerns us in the present work. The behavior of a double-diffusive system heated from the side was also described by Stern [11] and Thorpe et al. [12].

With lateral heating and a laterally imposed concentration gradient, convection develops without threshold except in a particular limit in which solute buoyancy is exactly compensated with thermal buoyancy, thus making possible the existence of a purely diffusive solution. This case was studied in [13] and then in detail in [14] for low Rayleigh number, establishing the nature of the first bifurcation from this quiescent state. A subsequent work [15] studied the dependence of the first bifurcation on the aspect ratio and in another work [16] the bifurcation diagram of the steady solutions for low Rayleigh number was computed for different inclinations of the cavity, finally connecting with the horizontal problem.

The genericity of this particular case of compensating gradients was tested in [17]. It was proved that the bifurcation diagram is generic, except for the purely diffusive state which is destroyed when we move away from the compensating gradients case. This system has been further studied in [18], where the influence of the third dimension is studied for different aspect ratios. In [19,20] the multiplicity of steady states in the large aspect ratio containers of Ref. [16] is studied and interpreted in terms of localized steady states (convectons).

In the case of lateral heating but without imposing a lateral concentration gradient a similar quiescent state can be obtained, in this case due to the solutal gradient built because of the Soret effect [2]. Note that this case is inequivalent to the previous one. Close to the threshold of the instability of the purely diffusive state, the behavior was found to be similar to the one found in the double-diffusive case. Nevertheless, as the Rayleigh number is increased, a richer variety of scenarios has been found. Most interestingly the dynamics is dominated by a low-frequency nonsymmetric periodic solution which appears in a global bifurcation, experiences multiple transformations and finally disappears in a blue sky catastrophe [2].

This behavior is in sharp contrast with the one found in the pure fluid case, for the same values of the Prandtl number [21,1]. In that case, for the same range of explored Rayleigh numbers, the dynamics remains symmetric and steady, and loses stability in a Hopf bifurcation to a short-period symmetric solution, which disappears for much higher values of the Rayleigh number in a very complex scenario leading to a chaotic attractor [1].

To explore the genericity of the results of [2] and to bridge the gap with the pure fluid case, a study of the successive transformations of the global connection that gives birth to the aforementioned low-frequency nonsymmetric periodic solution was undertaken in [3]. It was found that this global connection changed in nature from a saddle–loop to a SNIC and then again to a saddle–loop, finally to become a Hopf bifurcation in a Takens–Bogdanov (TB) point.

This Takens–Bogdanov point was found to be associated with a saddle–node-separatrix loop (SNSL) bifurcation and with a cusp. This conjunction of codimension-two bifurcations has been found in many problems of very different nature, from systems biology [22] to lasers [23]. In [3] it was found that this conjunction and all the low-Rayleigh number dynamics could be described as a part of the unfolding of a degenerate (codimension-three) Takens–Bogdanov point, the *saddle* case found by Dumortier et al. [24].

In this paper we present a comprehensive picture of the transformations that the bifurcation diagram undergoes as we move away from the case of compensating gradients to the case of pure fluid. To this end a wide range of variation of parameters (Rayleigh number and separation ratio) is explored, and different numerical techniques (mainly temporal integration of dynamical equations and continuation of steady solutions) are employed. In particular we pay a special attention to the disappearance of the long-period nonsymmetric solution and to the change of character of the bifurcations from global to local. This kind of investigation is hampered by the long temporal scales of the associated dynamics and the complexity of the different scenarios found, and indeed an arduous numerical effort has been necessary to complete the task. Moreover, the range of parameters spanned is quite large and some of the most interesting and complex dynamics occurs in very small regions of parameter space. Luckily, as in Ref. [3], the topology of the bifurcation lines could be rationalized as associations of codimension-two bifurcations that could be linked to higher codimension phenomena, which in fact resulted to be a guide to find and organize them. We think that such connections constitute an important result of this study.

The structure of the paper is as follows. In Section 2 we state the system under study, its equations and the numerical methods employed. In Section 3 we present the results for the different ranges of parameters explored, and discuss the structure of the bifurcation diagrams. Finally we present the conclusions in Section 4.

2. Formulation of the problem

Binary mixtures are characterized by a cross-diffusion effect called the Soret effect that describes the diffusive separation of the lighter and heavier components of the mixture in an imposed temperature gradient. Specifically, if c is the concentration of the heavier component, its flux is proportional to $-(\nabla c + c(1-c)S_T\nabla T)$, where S_T is the Soret coefficient, and T is the temperature; then, when S_T is negative the heavier component migrates, on a diffusive time scale, towards the hotter boundary.

In our work, we consider a binary mixture in a 2-D rectangular cavity of aspect ratio $\Gamma = d/h = 2$, where d is the length and h is the height of the cavity. A difference of temperature ΔT is maintained between the right and left vertical walls ($T_r - T_l = \Delta T$) and at the horizontal walls, which are assumed to be perfectly conducting, a linear temperature profile between the two prescribed temperatures is imposed. All walls are considered no-slip and impervious (no mass flux) boundaries.

The Boussinesq equations describing the system are nondimensionalized using ΔT as unit of temperature, h as the unit of length and the thermal diffusion time in the vertical direction $t_\kappa = h^2/\kappa$ as the unit of time, κ being the thermal diffusivity. In the equation of mass-conservation we approximate the expression of the flux of the heavier component substituting the concentration c appearing in the thermodiffusion term by its concentration c_0 in the homogeneous mixture and thus the concentration field is scaled by $\Delta c = -c_0(1-c_0)S_T\Delta T$.

With the stated approximations the dimensionless equations explicitly read

$$\partial_t \mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla P + \sigma\nabla^2\mathbf{u} + \sigma Ra[(1+S)(-0.5+x/\Gamma) + \theta + SC]\hat{\mathbf{z}}, \quad (1)$$

$$\partial_t\theta + (\mathbf{u}\cdot\nabla)\theta = -v_x/\Gamma + \nabla^2\theta, \quad (2)$$

$$\partial_t C + (\mathbf{u}\cdot\nabla)C = -v_x/\Gamma - \tau\nabla^2(\theta - C), \quad (3)$$

$$\nabla\cdot\mathbf{u} = 0, \quad (4)$$

where $\mathbf{u} \equiv (v_x, v_z)$ is the velocity field in (x, z) coordinates, P is the pressure over the density, and θ denotes the dimensionless departure of the temperature from a linear horizontal profile. C is the scaled deviation of the concentration of the heavier component relative to the linear horizontal profile which would develop in a pure diffusive state. The dimensionless parameters are the Prandtl number $\sigma = \nu/\kappa$, the Rayleigh number $Ra = \alpha g h^3\Delta T/\nu\kappa$, the