SEPARABILITY OF MULTIPARTITE QUANTUM SYSTEMS

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A necessary condition for separability of multipartite quantum systems is given. It is shown that for $2 \times 2 \times \cdots \times 2 \times d$ quantum systems the condition of separability is equivalent to the criterion of positive partial transposition. We also define an entanglement measurement based on this separability condition which can be considered as a kind of generalization of negativity.

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Entanglement is an essential ingredient in quantum information and a central feature of quantum mechanics which distinguishes a quantum system from its classical counterpart. As an important physical resource, it is also widely applied to a lot of quantum information processing (QIP): quantum computation [1], quantum cryptography [2], quantum teleportation [3], quantum dense coding [4], and so on.

However, the most basic problems in quantum entanglement theory such as the separability of arbitrary multipartite quantum states or the measurement of multipartite entanglement are still unsolved. Here we focus on the separability of quantum states. Recently considerable amount of work was done to derive separability criterions for multipartite quantum states. For multipartite pure states, the generalized concurrence given in [5] can be used to judge if the state is separable or not. For multipartite mixed states, there are separability criteria such as PPT, realignment etc. [6, 7]. For tripartite case, we have derived a necessary condition for separability in [8]. In [9], the authors give a separability criterion useing the Bloch representation of multipartite quantum states and the algebra of higher-order tensors.

In this paper, we continue the research in [8] to give a general separability criterion for arbitrary multipartite quantum systems. The paper is organized as follows: First, we give a short review of the Bloch sphere representation of a multipartite quantum state with arbitrary dimension. Secondly, we generalize the criterion from [8] for separability of tripartite quantum state to multipartite systems. Thirdly, we define an entanglement measurement according to this separability criterion by applying the formalism presented in [10]. The newly defined entanglement measurement can be considered as a kind of generalization of the negativity.

Before embarking on this study, we first give a short review of the tripartite separability criterion. In [8] we have pointed out that any tripartite quantum state $\rho_{ABC} \in \mathcal{H}_{N_1} \otimes \mathcal{H}_{N_2} \otimes \mathcal{H}_{N_3}$ can be written as

$$\rho_{ABC} = I_{N_1} \otimes I_{N_2} \otimes M_0 + \sum_{i=1}^{N_1^2-1} \lambda_i(1) \otimes I_{N_2} \otimes M_i + \sum_{j=1}^{N_2^2-1} I_{N_1} \otimes \lambda_j(2) \otimes \widetilde{M}_j + \sum_{i=1}^{N_1^2-1} \sum_{j=1}^{N_2^2-1} \lambda_i(1) \otimes \lambda_j(2) \otimes M_{ij}, \quad (1)$$

where $\lambda_i(1)$, $\lambda_j(2)$ are generators of $SU(N_1)$ and $SU(N_2)$; $M_i, \widetilde{M}_j$ and M_{ij} are operators of $\mathcal{H}_{N_3}$ defined by

$$\begin{aligned} M_0 &= \frac{1}{4}\mathrm{Tr}_{AB}(\rho_{ABC}), \\ M_i &= \frac{1}{4}\mathrm{Tr}_{AB}(\rho_{ABC}\lambda_i(1) \otimes I_{N_2} \otimes I_{N_3}), \\ \widetilde{M}_j &= \frac{1}{4}\mathrm{Tr}_{AB}(\rho_{ABC} I_{N_1} \otimes \lambda_j(2) \otimes I_{N_3}), \\ M_{ij} &= \frac{1}{4}\mathrm{Tr}_{AB}(\rho_{ABC}\lambda_i(1) \otimes \lambda_j(2) \otimes I_{N_3}); \end{aligned}$$

and N_1, N_2 and N_3 are dimensions of the systems A, B and C respectively.

Let $R(1)$, $R(2)$ be $(N_1^2 - 1) \times (N_1^2 - 1)$, $(N_2^2 - 1) \times (N_2^2 - 1)$ real matrices satisfying

$$\frac{1}{(N_1 - 1)^2} I - R(1)^T R(1) \geq 0$$

and

$$\frac{1}{(N_2 - 1)^2} I - R(2)^T R(2) \geq 0,$$

respectively. Then a new operator $\gamma_{\mathcal{R}}$ can be constructed by defining

$$\gamma_{\mathcal{R}}(\rho_{ABC}) = I_{N_1} \otimes I_{N_2} \otimes M_0' + \sum_{i=1}^{N_1^2-1} \lambda_i(1) \otimes I_{N_2} \otimes M_i' + \sum_{j=1}^{N_2^2-1} I_{N_1} \otimes \lambda_j(2) \otimes \widetilde{M}_j' + \sum_{i=1}^{N_1^2-1} \sum_{j=1}^{N_2^2-1} \lambda_i(1) \otimes \lambda_j(2) \otimes M_{ij}', \quad (2)$$