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# Large fluctuations of the nonlinearities in isotropic turbulence. Anisotropic filtering analysis

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## HIGHLIGHTS

- The article highlights the difference between Navier–Stokes and Kolmogorov turbulence in terms of stretching–tilting statistics.
- Vortex stretching magnitude normalized over the local enstrophy $= f$.
- Near zero probability for stretching of intensity larger than twice the enstrophy.
- Anisotropic filtering proposed for targeting different structure kinds.
- Inertial range blobs filtered out: $f$ increases; larger ones filtered out: $f$ falls.

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## ABSTRACT

Using a Navier–Stokes isotropic turbulent field numerically simulated in a box with a discretization of $1024^3$ (Biferale et al., 2005), we show that the probability of having a stretching–tilting larger than a few times the local enstrophy is low. By using an anisotropic kind of filter in the Fourier space, where wavenumbers that have at least one component below a threshold or inside a range are removed, we analyze these survival statistics when the large, the small inertial or the small inertial and dissipation scales are filtered out. By considering a flow obtained by randomizing the phases of the Fourier modes, and applying our filtering techniques, we identified clearly the properties attributable to turbulence.

It can be observed that, in the unfiltered isotropic Navier–Stokes field, the probability of the ratio $(|\boldsymbol{\omega}\cdot\nabla\mathbf{U}|/|\boldsymbol{\omega}|^2)$ being higher than a given threshold is higher than in the fields where the large scales were filtered out. At the same time, it is lower than in the fields where the small inertial and dissipation range of scales is filtered out. This is basically due to the suppression of compact structures in the ranges that have been filtered in different ways. The partial removal of the background of filaments and sheets does not have a first order effect on these statistics. These results are discussed in the light of a hypothesized relation between vortical filaments, sheets and blobs in physical space and in Fourier space. The study in fact can be viewed as a kind of test for this idea and tries to highlight its limits. We conclude that a qualitative relation in physical space and in Fourier space can be supposed to exist for blobs only. That is for the near isotropic structures which are sufficiently described by a single spatial scale and do not suffer from the disambiguation problem as filaments and sheets do.

Information is also given on the filtering effect on statistics concerning the inclination of the strain rate tensor eigenvectors with respect to vorticity. In all filtered ranges, eigenvector 2 reduces its alignment, while eigenvector 3 reduces its misalignment. All filters increase the gap between the most extensional eigenvalue $\langle\lambda_1\rangle$ and the intermediate one $\langle\lambda_2\rangle$ and the gap between this last $\langle\lambda_2\rangle$ and the contractile eigenvalue $\langle\lambda_3\rangle$. When the large scales are missing, the modulus of the eigenvalue 1 becomes nearly equal to that of the eigenvalue 3, similarly to the modulus of the associated components of the enstrophy production.



## 1. Introduction

The formation of spatial and temporal internal scales can in part be associated to the stretching and tilting of vortical structures. Many aspects of the behavior of turbulent fields have been

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associated to this phenomenon: the onset of instability, vorticity intensification or damping, production and dissipation or the three-dimensionalization of the flow field [1–5]. In the standard picture of turbulence, the energy cascade to smaller scales is interpreted in terms of the stretching of vortices due to the interaction with similar eddy size (see for example [6]). A number of statistical details on the stretching phenomenon and the closely related enstrophy production can be found in the monography by Tsinober (2001, see in particular Chapter 6, [7]).

Although the important physical role of these inertial phenomena is recognized, the literature does not often include statistical information on quantities such as the magnitude or the components of $\boldsymbol{\omega} \cdot \nabla \mathbf{U}$. For instance, in a letter to Nature (2003) dedicated to the measurements of intense rotation and dissipation in turbulent flows, Zeff et al. [8] observe that the understanding of the temporal interactions between stretching and vorticity is crucial to the science of extreme events in turbulence. However, the statistics presented there concern dissipation and enstrophy and not directly stretching. The literature more often includes statistical information concerning other gradient quantities such as the strain rate or the rate-of-rotation tensors, and, in particular, their fundamental constituents: the longitudinal or transverse velocity derivatives. The skewness and flatness factors of the velocity derivative have been considered in a number of laboratory and numerical studies. For instance, Batchelor and Townsend in 1949 [9], through the study of the oscillograms of the velocity derivatives, showed that the energy associated with large wave numbers is very unevenly distributed in space. More recently it has been shown how velocity derivatives increase monotonically with the Reynolds number, see e.g. [10,11], and the reviews by Sreenivasan and Antonia (1997) [12] and Ishihara, Gotoh and Kaneda (2009) [13]. In particular, Ishihara et al. [14], considering one-point statistics of velocity gradients and Eulerian and Lagrangian accelerations analyzed the data from high-resolution direct numerical simulations (DNS) of turbulence in a periodic box, with up to $4096^3$ grid points, and found for these gradients an algebraic dependence on $Re_\lambda$.

One-point statistics of velocity gradients and Eulerian and Lagrangian accelerations are studied by analyzing the data from high-resolution direct numerical simulations (DNS) of turbulence in a periodic box, with up to $4096^3$ grid points.

In the case of turbulent wall flows, laboratory measurements of both the mean and the r.m.s. of fluctuations of the stretching components across the two-dimensional boundary layer have been reported by Andreapoulos and Honkan (2001) [15]. In this study, the normalized r.m.s values of the stretching components are very significant throughout the boundary layer and reach values that are one order of magnitude larger than the mean span-wise component (the only significant mean component, however and only in the near wall region). The values observed for the r.m.s. of the stretching range from 0.04, close to the wall, to about 0.004 in the outer part.

In a study concerning the structure and dynamics of vorticity and rate of strain in incompressible homogeneous turbulence, Nomura and Post (1998) [16] demonstrate the significance of both local dynamics (influence of local vorticity) and spatial structure (influence through non-local pressure Hessian) in the interaction of the vorticity and strain rate tensor. The behavior of high-amplitude rotation-dominated events cannot be solely represented by local dynamics due to the formation of distinct spatial structure. Instead, high-amplitude strain dominated regions are generated predominantly by local dynamics. The associated structure is less organized and more discontinuous than the one associated with rotation dominated events. They conclude that non-local effects are significant in the dynamics of small scale motion. This should be considered in the interpretation of single-point statistics. Characterizations of small-scale turbulence should consider not only the typical structures there present but also typical structure interactions. In this context these authors offer the radial distribution of the magnitude of the strain rate tensor normalized on the enstrophy. In this paper the maximum value of this magnitude is found close to 0.2.

Laboratory statistical information on the stretching of field lines can be found in [17]. Here, probability density functions of the logarithm of the local stretching in *N* cycles were obtained for several two-dimensional time-periodic confined flows exhibiting chaotic advection. The stretching fields were observed to be highly correlated in space when *N* is large, and the probability distributions were observed to be similar for different flows.

However, a few examples in literature can also be cited regarding direct results for stretching–tilting statistics. For instance, recently experimental and numerical confirmation has been found of the predominance of three dimensional turbulent vortex stretching in the positive net enstrophy production. These aspects have been extensively considered in Tsinober (2000) [18] and in the 2001 monography [7], where a number of statistical geometrical details concerning the vortex alignment, compression, tilting, and folding are outlined. Through two papers, Constantin, Procaccia and Segel (1995) [19], Galanti, Procaccia and Segel (1996) [20] consider the stretching and its relationships with the amplification of vorticity and the straightening of the vortex lines. They show that the same stretching that amplifies the vorticity also tends to straighten out the vortex lines. They also show that in well-aligned vortex tubes, the self-stretching rate of the vorticity is proportional to the ratio of the vorticity and the radius of curvature. In this context [20] gives statistics on the stretching and vortex line curvature. Numerically this is seen as the appearance of high correlations between the stretching and the straightness of the vortex lines. Regarding to this issue, an important universal feature of fully developed turbulent flows is the preferential alignment of vorticity along the eigendirection of the intermediate eigenvalue of the strain-rate tensor. A number of works both experimental and numerical studies on this result are available (Tsinober, Kit and Dracos (1992) [10], Kholmyansky, Tsinober and S. Yorish (2001) [21], Gulitski et al. (2007) [22–24] and Chevillard et al. (2008) [25]). It should be noticed, however, that in the case of *nonlocal* strain rate, Hamlington, Schumacher and Dahm [26], have observed a direct assessment of vorticity alignment with the most extensional eigenvector by using data from highly resolved direct numerical simulations.

In the present study, for the case of isotropic turbulence ($Re_\lambda = 280$ [27]), we consider statistics related to the intensity of the stretching term in the equation for vorticity. If we consider the general instantaneous local intrinsic anisotropy of turbulent fields, looking at stretched structures as filaments and sheets, we would like to be able to disentangle them to follow and understand better their evolution and detailed dynamics. Isotropic filtering is unable to carry out this job.

We have conceived a probe function, the ratio between the magnitude of the vortex stretching and the enstrophy, to empirically and statistically measure the local activity of the stretching phenomenon (see Section 2). In addition, we propose an alternative to the commonly used isotropic filter: the cross filter. This is a new empirical, and at the moment limited, attempt to introduce an anisotropic filtering. In Section 3, we analyze the survival function of the normalized stretching by using the cross filter acting directly on the velocity Fourier space. We do this in the hope of qualitatively highlighting aspects related to the role of the three-dimensional structures known as blobs, sheets and filaments and their hypothetical Fourier counterparts. This study can be viewed as a kind of test for this idea and tries to highlight its limits. To check the implication of the filtering analysis, we quantified for each estimator considered an approximately Gaussian reference velocity field