STARK RESONANCES IN 2-DIMENSIONAL CURVED QUANTUM WAVEGUIDES

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In this paper we study the influence of an electric field on a two-dimensional waveguide. We show that bound states that occur under a geometrical deformation of the guide turn into resonances when we apply an electric field of small intensity having a nonzero component on the longitudinal direction of the system.

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1. Introduction

The study of resonances occurring in a quantum system subject to a constant electric field is now a well-known issue among the mathematical physics community. In a recent past a large amount of literature has been devoted to this problem (see e.g. [15, 17] and references therein). Mostly these works are concerned with quantum systems living in the whole space $\mathbb{R}^n$ as e.g. atomic systems [6, 12, 16, 18, 26, 27]. In the present paper we would like to address this question for an inhomogeneous quantum system existing in a curved quantum waveguide in $\mathbb{R}^2$. It is known that bound states arise in curved guides [7, 10] and the corresponding eigenfunctions are expected to be localized in space around the deformation. Therefore, based on these results the main question is what happens with these bound states when the electric field of small intensity is switched on?

A first result is given in [11] where the electric field is supposed to be orthogonal to the guide outside a bounded region. But in this situation there is no Stark resonance.

Here we are focusing on a strip $\Omega \subset \mathbb{R}^2$ of constant width curved within a compact region. The electric field is chosen with a strictly positive component both in the longitudinal direction of the left part and of the right part of the curved strip. Roughly speaking, this situation is similar to the one of an atomic system interacting with an external electric field. Due to the field, an eigenstate of the curved waveguide at zero field turns into scattering state which is able to escape at infinity under the dynamics. It is then natural to expect spectral resonances for this system. In this work we would like to study this question in the weak field regime.

The resonances are defined as the complex poles in the second Riemann sheet of the meromorphic continuation of the resolvent associated to the Stark operator. We construct this extension using the distortion theory [4, 19]. Our proof of existence of resonances borrows elements of strategy developed in [6, 16]. It is mainly based on nontrapping estimates of [6]. For the applicability of these techniques to our model, the difficulty we have to solve is that the system has a bounded transverse direction.

To end this section let us mention a still open question related to this problem and that we hope to solve in a future work. We claim that our regularity assumptions on the curvature imply that the corresponding Stark operator (see (2.4)) has no real eigenvalue [5]. In that case the complex poles have a nonzero imaginary part, then they are resonances in the strict sense of the term [24].

Let us briefly review the content of the paper. In Section 2 we describe precisely the system, assumptions and the main results. The distortion and the definition of resonances are given respectively in Sections 3 and 4. In Section 5 we prove the existence of resonances. Finally, Section 6 is devoted to get an exponential estimate on the width of resonances. Actually we show that the imaginary part of resonances arising in this system follows a type of Oppenheimer's law [22] when the intensity of the field vanishes.

2. Main results

2.1. Setting

Before describing the main results of the paper we want to recast the problem into a more convenient form. This allows us to state precisely our assumptions on the system.

Consider a curved strip Ω in $\mathbb{R}^2$ of a constant width d defined around a smooth reference curve Γ, we suppose that Ω is not self-intersecting. The points $\mathbf{X} = (x, y)$ of Ω are described by the curvilinear coordinates $(s, u) \in \mathbb{R} \times (0, d)$,

$$\begin{aligned} x &= a(s) - ub'(s), \\ y &= b(s) + ua'(s), \end{aligned} \tag{2.1}$$

where a, b are smooth functions defining the reference curve $\Gamma = \{(a(s), b(s)), s \in \mathbb{R}\}$ in $\mathbb{R}^2$. They are supposed to satisfy $a'(s)^2 + b'(s)^2 = 1$.

Introduce the signed curvature $\gamma(s)$ of Γ,

$$\gamma(s) = b'(s)a''(s) - a'(s)b''(s). \tag{2.2}$$