STOKES' FIRST PROBLEM FOR A THERMOELECTRIC FLUID WITH FRACTIONAL-ORDER HEAT TRANSFER

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In this work, the flow for thermoelectric fluid over a suddenly moving heated plane surface using the methodology of fractional calculus is examined. The resulting formulation is applied to Stokes' first problem subjected to an arbitrary heating which is taken as a function of time. Using Laplace transforms and direct approach, the exact solutions are obtained in Laplace domain by a simple method. Numerical results are computed for two cases and represented graphically. Some comparative diagrams concerning the temperature and velocity component profiles are presented.

Keywords: magnetohydrodynamic, thermoelectric fluid, Stokes' first problem, Laplace transforms, fractional calculus, numerical results.

1. Introduction

Physical observations and results of the conventional coupled thermoelectric fluid mechanics theories involving infinite speed of thermal signals which were based on the mixed parabolic-hyperbolic governing equations of Shercliff [1] are mismatched. To remove this paradox, the conventional theories of thermoelectric fluid have been generalized, where the generalization is in the sense that these theories involve a hyperbolic-type heat transport equation supported by experiments which exhibit the actual occurrence of wave-type heat transport in solids called second sound effect. The first is due to Cattaneo [2] who obtained a wave-type heat equation by postulating a new law of heat conduction replacing the classical Fourier law.

Stokes in 1851 and again Rayleigh in 1911 discussed the fluid motion above the plate independently taking the fluid to be Newtonian [3]. Due to the variety of fluids, various researchers extended Stokes' problem with different conditions and fluid models, for example, Teipel [4] for a second-grade fluid, Morrison [5] for a Jeffrey fluid with no small viscosity, Huilgol [6] for a fluid with small viscosity, Tanner [7] for a viscoelastic fluid, Preziosi and Joseph [8] for viscoelastic fluids with a viscosity and relaxation kernel, Phan-Thien and Chew [9] for a modified Phan-Thien–Tanner model and for Rivlin–Ericksen fluids [10–14] . Recently, Devakar and Iyengar [15] studied Stokes' first problem for a micropolar fluid using the state-space approach [16].

Stokes' first problem has in recent years received much attention due to its practical applications. Vít Průša [17] revisited the numerical study carried out by Srinivasan and Rajagopal [18] and he showed that the long time solutions to the first and second Stokes problem for fluids with pressure dependent viscosities can be given by exact formulae. Muzychka and Yovanovich [19] examined three problems: unsteady Couette flow, unsteady Poiseuille flow, and unsteady boundary layer flow. Chen et al. [20] developed the numerical methods with fourth-order spatial accuracy for variable-order nonlinear Stokes' first problem for a heated generalized second grade fluid.

The performance of thermoelectric devices depends heavily on the material intrinsic property, Z, known as the figure of merit and defined by $Z = \sigma_o S^2/\kappa$ where σ, κ and S are respectively the electrical conductivity, thermoelectric power, and thermal conductivity. Increasing of the parameter Z has a positive effect on the efficiency of thermoelectric device.

The interaction between the thermal and magnetohydrodynamic fields is a mutual one owing to alterations in the thermal convection and to the Peltier and Thomson effects ($P = ST$ [21], where P is a Peltier coefficient, although these are usually small). Thermoelectric devices have many attractive features compared with the conventional fluid-based refrigerators and power generation technologies, such as long life, no moving part, no noise, easy maintenance and high reliability. However, their use has been limited by the relatively low performance of the present thermoelectric materials [22].

Differential equations of fractional order have been the focus of many studies due to their frequent appearance in various applications in fluid mechanics, viscoelasticity, biology, physics, and engineering. The differential equations involving Riemann–Liouville differential operators of fractional order $0 < \alpha < 1$ appear to be important in modelling several physical phenomena [23] and therefore seem to deserve an independent study of their theory parallel to the well-known theory of ordinary differential equations. The first application of fractional derivatives was given by Abel who applied fractional calculus in the solution of an integral equation that arises in the formulation of the tautochrone problem. One can state that the whole theory