# COHOMOLOGICAL RESOLUTIONS FOR ANOMALOUS LIE CONSTRAINTS

ZBIGNIEW HASIEWICZ and CEZARY J. WALCZYK

Department of Physics, University of Białystok, ul. Lipowa 41, 15-424 Białystok, Poland
(e-mails: z.hasiewicz@gmail.com, c.walczyk@alpha.uwb.edu.pl)

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It is shown that the BRST resolution of the spaces of physical states of the systems with anomalies can be consistently defined. The appropriate anomalous complexes are obtained by canonical restrictions of the ghost extended spaces to the kernel of anomaly operator without any modifications of the "matter" sector. The cohomologies of the anomalous complex for the case of constraints constituting a centrally extended simple Lie algebra of compact type are calculated and analyzed in details within the framework of Hodge–deRham–Kähler theory: the vanishing theorem of the relative cohomologies is proved and the absolute cohomologies are reconstructed.

**Keywords:** constrained quantum systems, cohomology, anomaly, curvature, vanishing theorem, Hodge–deRham–Kähler theory.

## Introduction

The cohomological approach to constrained systems or systems with gauge symmetries was initiated in the seventies of the last century [1]. Since that time it had grown into quite advanced and powerful machinery with successful applications in field theory as well as in string theory [2]. It is still under investigation and development on the classical and quantum levels. The cohomological BRST formalizm appeared to be very efficient tool to describe the interactions of fields and/or strings.

The BRST cohomological approach is well established for first class [3] systems of constraints. Most of the interesting physical systems are governed by the constraints of mixed type. Its generalization to the systems of mixed type is not unique and there are several approaches. One of the proposals is to solve all constraints of second class already on the classical level in order to obtain the first class classical system to be quantized. This approach has two important drawbacks. First of all, it might appear that upon quantization the first class system gets quantum anomaly (which happens mainly in the case of infinitely many degrees of freedom) as in field theory or string theory. Secondly, one might obtain completely inadequate picture of the system at quantum level. The simplest example which comes into mind is a particle interacting with a centrally symmetric potential (eg. hydrogen atom) with constraint which fixes one of its angular momentum at nonzero value. The reduction of this system on the classical level leads after canonical quantization to "flat" picture of its states and to wrong spectrum of angular momentum upon quantization. The appropriate reduction on the quantum level based on the Gupta–Bleuler [4] polarization of constraints leads to a different and consistent result. The remark based on this simple example leads one to the conclusion that the diagram

$$\begin{array}{ccccc}
\mathcal{C} & \xrightarrow{\text{quantization}} & & & \mathcal{Q} \\
\text{classical reduction}\big\downarrow & & & & \big\downarrow\text{GB quantum reduction} \\
\mathcal{C}_{\text{red}} & \xrightarrow{\text{quantization}} & \mathcal{Q}'_{\text{red}} & \stackrel{?}{\leftrightarrow} & \mathcal{Q}_{\text{red}}
\end{array}$$

cannot be converted into commutative one, as it seems that an appropriate map marked by "?" cannot be consistently defined.[1] What is most important in the above example: the way of proceeding according to Gupta–Bleuler rules [4], indicated by the right-hand side of the diagram, gives the physically acceptable result in agreement with common intuition and knowledge. One may also think about constrained quantum system without any relation to underlying classical one as it happened with Dual Theory [6] based on the axiomatic approach to S-matrix. The next examples, which evidently indicate that the above diagram cannot be converted into commutative one are given by the models of critical massive strings and non critical massless strings [7].

For this reason the approach based on the polarization of the quantum constraints, which allows one to proceed with an equivalent system of first class at the quantum level seems to be reasonable.

The situation is more or less standard if the algebra of constraints admits a real polarization—which is rather rarely encountered case within the class of the physical systems of importance. The problem becomes far from being obvious when the polarization is necessary complex. This last case includes the most important physical theories and models: quantum electrodynamics [4], noncritical string theories [7] and high-spin systems [8].

There were some early proposals how to treat the constrained system in this situation [9] but that approach was prematured and far from being canonical and consistent.

The canonical and mathematically consistent approach to cohomological BRST description of constrained systems of mixed class is proposed in this paper. Although it was grown on the backgrounds of the experience in string theory [10] and high-spin systems [8] the authors are convinced that its main ideas and results are universal as the underlying constructions can be easily (neglecting technical difficulties) extended to the wide class of models of physical importance. For this reason and in order to avoid technical difficulties, which would screen the main ideas, the authors

[1]The vertical arrow on the left-hand side of the diagram is strictly defined [5] witin the framework of symplectic geometry