Physica D 241 (2012) 1169–1178

Contents lists available at SciVerse ScienceDirect

Physica D

journal homepage: www.elsevier.com/locate/physd

Novel bifurcation structure generated in piecewise-linear three-*LC* resonant circuit and its Lyapunov analysis

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ARTICLE INFO

Article history:
Received 16 April 2011
Received in revised form
22 March 2012
Accepted 24 March 2012
Available online 13 April 2012
Communicated by G. Stepan

Keywords:
Lyapunov analysis
Two-torus
Three-torus
Hyperchaos

ABSTRACT

We analyse a piecewise-linear oscillator that consists of a three-*LC* resonant circuit with a hysteresis element. Three sets of two-dimensional linear equations, including a hysteresis function, represent the governing equations of the circuit, and all the Lyapunov exponents are calculated in a remarkably simple manner based on derived explicit solutions. Various dynamical phenomena such as two-torus, three-torus, and hyperchaos with four positive Lyapunov exponents are observed by Lyapunov analysis. We obtained a detailed bifurcation diagram in which novel bifurcation structure which we call a "two-torus Arnold tongue" is observed where two-torus generating regions exist in a three-torus generating region as if periodic states exist in a two-torus generating region.

1. Introduction

In this paper, we report a novel bifurcation structure of a piecewise-linear three-*LC* resonant circuit in which complicated dynamical phenomena are observed. We analyse Lyapunov exponent of the system and draw a bifurcation diagram that shows hyperchaos, two-torus, and three-torus.

Because of the recent substantial increase in computational power, the detailed analyses of higher-dimensional ordinary differential equations (ODEs), partial differential equations (PDEs), and delay differential equations (DDEs) have become possible, where a rich variety of interesting phenomena such as the two-torus and three-torus, chaos, and hyperchaos are observed [1–7]. Although a numerical integration method is widely used in the analysis of nonlinear dynamical systems, we have to be cautious regarding numerical errors, especially in higher-order dynamical systems, because chaos is a sensitive phenomenon in many cases [8–10]. A piecewise-linear technique is a powerful method to overcome these errors [11–13]. In the piecewise-linear oscillators, an explicit solution is obtained in each piecewise-linear branch. Therefore, numerical errors can be decreased in such systems.

However, applying a piecewise-linear technique to higher-dimensional systems is difficult. Hosokawa and Nishio attempted the analysis of a piecewise linear sixth-order chaos-generating circuit [14]. The dynamics is represented by the six-dimensional differential piecewise linear equations and its characteristic equation is a six-degree algebraic equation. However, we do not have formulae for solving algebraic equations of five- or more degrees. Hence, the characteristic equation has to be solved numerically. In addition, it must be noted that the procedure to derive all Lyapunov exponents for such higher-dimensional dynamical systems might be difficult and impractical. In [14], for example, only the largest Lyapunov exponent was calculated.

In this study, we conduct Lyapunov analysis of a three-*LC* resonant circuit with a hysteresis element, which is an extended version of "a four-dimensional plus hysteresis chaos generator" proposed by Mitsubori and Saito [15]. Their circuit consists of two linear negative resistance *LC* oscillators, which are connected by one hysteresis element. The linear negative resistances supply energy, and the hysteresis element consumes the energy. Mitsubori and Saito analysed a four-dimensional case. Since a formula for a

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doi:10.1016/j.physd.2012.03.011

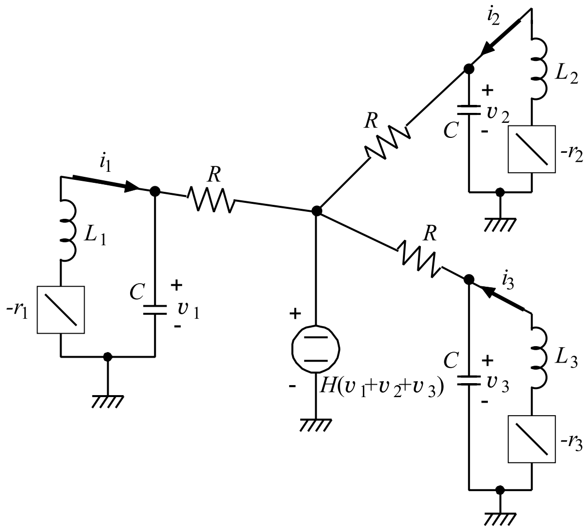

Fig. 1. Circuit diagram.

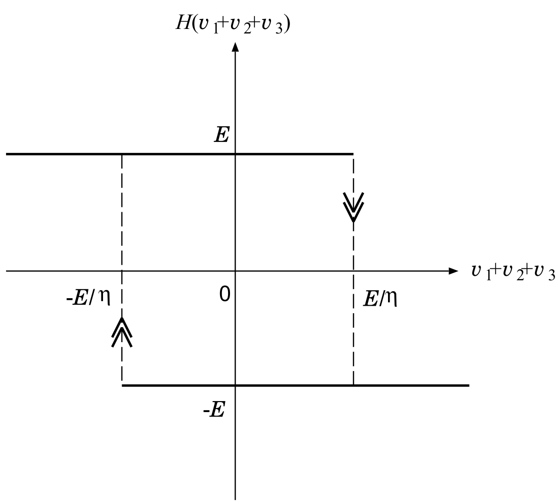

Fig. 2. Characteristic of hysteresis.

four-degree algebraic equation exists, it was possible to solve the four-dimensional piecewise-linear dynamical systems. We consider herewith a problem to analyse a higher-dimensional system of a chaos generator, where no formula is available for solving the characteristic equation.

The merits of the analysis of the three-*LC* resonant circuit are as follows. The system is represented by three two-dimensional simultaneous equations, including a hysteresis function, and thus, it is not necessary to solve a six-dimensional algebraic equation. Alternatively, we solve three two-dimensional algebraic equations. Poincaré mapping is five-dimensional, and all the Lyapunov exponents can be calculated in a remarkably simple manner using derived explicit solutions.

Finally, we analyse a detailed bifurcation diagram in which three-torus, two-torus and chaotic solutions are observed in a wide range of system parameters. In addition, hyperchaos with four positive Lyapunov exponents is observed. They exist in the bifurcation diagram in a complicated manner. In particular, a noteworthy bifurcation structure which we call "two-torus Arnold tongue" is found where two-torus generating regions exist in a three-torus generating region.

2. Circuit set-up and its normal form

Fig. 1 illustrates the circuit diagram considered in this paper, where $-r_1$, $-r_2$, and $-r_3$ are linear negative resistances, and H is a hysteresis element whose output voltage is controlled by $v_1 + v_2 + v_3$. Fig. 2 shows the hysteresis characteristic represented by the following equations:

$$H(v_1+v_2+v_3) = \begin{cases} -E & \text{for } v_1+v_2+v_3 \geq -E/\eta \\ E & \text{for } v_1+v_2+v_3 \leq E/\eta. \end{cases} \tag{1}$$

The governing equation of the circuit is represented by simultaneous sixth-order equations, including a hysteresis function as follows:

$$\begin{aligned} RC\frac{\mathrm{d}v_1}{\mathrm{d}t} &= Ri_1 - (v_1 - H(v_1+v_2+v_3)), \\ L_1\frac{\mathrm{d}i_1}{\mathrm{d}t} &= -v_1 + r_1 i_1, \\ RC\frac{\mathrm{d}v_2}{\mathrm{d}t} &= Ri_2 - (v_2 - H(v_1+v_2+v_3)), \\ L_2\frac{\mathrm{d}i_2}{\mathrm{d}t} &= -v_2 + r_2 i_2, \\ RC\frac{\mathrm{d}v_3}{\mathrm{d}t} &= Ri_3 - (v_3 - H(v_1+v_2+v_3)), \\ L_3\frac{\mathrm{d}i_3}{\mathrm{d}t} &= -v_3 + r_3 i_3. \end{aligned} \tag{2}$$

Then, let us rewire Eq. (2) by changing variables and parameters as follows:

$$\begin{aligned} &\tau' = t/RC, && \cdot = \mathrm{d}/\mathrm{d}\tau', \\ &X_1 = \eta v_1/E, && Y_1 = \eta R i_1/E, \\ &X_2 = \eta v_2/E, && Y_2 = \eta R i_2/E, \\ &X_3 = \eta v_3/E, && Y_3 = \eta R i_3/E, \\ &\alpha_1 = R^2C/L_1, && \beta_1 = r_1/R, \\ &\alpha_2 = R^2C/L_2, && \beta_2 = r_2/R, \\ &\alpha_3 = R^2C/L_3, && \beta_3 = r_3/R. \end{aligned} \tag{3}$$

The governing equation (2) is represented as follows:

$$\begin{aligned} \begin{pmatrix}\dot{X}_1\\ \dot{Y}_1\end{pmatrix} &= \begin{pmatrix}-1 & 1\\ -\alpha_1 & \alpha_1\beta_1\end{pmatrix}\left(\begin{pmatrix}X_1\\ Y_1\end{pmatrix} - \eta\begin{pmatrix}p_1\\ p_1/\beta_1\end{pmatrix} h(X_1+X_2+X_3)\right) \\ \begin{pmatrix}\dot{X}_2\\ \dot{Y}_2\end{pmatrix} &= \begin{pmatrix}-1 & 1\\ -\alpha_2 & \alpha_2\beta_2\end{pmatrix}\left(\begin{pmatrix}X_2\\ Y_2\end{pmatrix} - \eta\begin{pmatrix}p_2\\ p_2/\beta_2\end{pmatrix} h(X_1+X_2+X_3)\right), \\ \begin{pmatrix}\dot{X}_3\\ \dot{Y}_3\end{pmatrix} &= \begin{pmatrix}-1 & 1\\ -\alpha_3 & \alpha_3\beta_3\end{pmatrix}\left(\begin{pmatrix}X_3\\ Y_3\end{pmatrix} - \eta\begin{pmatrix}p_3\\ p_3/\beta_3\end{pmatrix} h(X_1+X_2+X_3)\right) \end{aligned} \tag{4}$$