ON THE LOCALIZED WAVE PATTERNS SUPPORTED BY CONVECTION-REACTION-DIFFUSION EQUATION

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A set of travelling wave solutions to convection-reaction-diffusion equation is studied by means of methods of local nonlinear analysis and numerical simulation. The occurrence is shown of the compactly supported solutions, shock fronts, and solitary waves for wide range of parameter values.

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1. Introduction

We consider the evolutionary equation,

$$u_t + u\,u_x - \kappa\,\left(u^n\,u_x\right)_x = (u - U_1)\,\varphi(u), \tag{1}$$

where κ, U_1 are positive constants, $\varphi(u)$ is a smooth function nullifying at the origin and nonzero within the set $(0, U_1]$. Eq. (1), referred to as convection-reaction-diffusion equation, is used to simulate transport phenomena in active media. Owing to its practical applicability and a number of unusual features, Eq. (1) was intensely studied in the recent decades [1–8].

The present investigations are mainly devoted to qualitative and numerical study of the family of travelling wave (TW) solutions of Eq. (1). Our objective is to show that, under certain conditions, the set of TW solutions contains periodic regimes, solitary waves, shock fronts and compactons. Solitary wave solutions (or solitons) are exponentially localized wave packets moving with constant velocity without changing their shape. They are mostly associated with the Korteveg–de Vries (KdV) equation and members of KdV hierarchy having the form

$$K(m) = u_t + u^m\,u_x + u_{xxx} = 0, \qquad m \geq 1. \tag{2}$$

For $m = 1$ the solitary wave solution of Eq. (2) is as follows [9],

$$u(t, x) = U(\xi) \equiv U(x - V\,t) = \frac{3\,V}{\cosh\left(\frac{\sqrt{V}}{2}\,\xi\right)},$$

where V stands for the wave packet velocity.

In 1993, Philip Rosenau and John Hyman put forward the following generalization to KdV hierarchy [10],

$$K(m, n) = u_t + (u^m)_x + (u^n)_{xxx} = 0, \qquad m, n \geq 2. \tag{3}$$

The modification of the higher-derivative term causes Eq. (3) to possess solutions with compact support. For $m = n = 2$, the compactly supported solution (or *compacton*) has the form

$$u(t, x) = U(\xi) \equiv U(x - V\,t) = \begin{cases} \frac{4\,V}{3} \cos^2\left[\frac{x - V\,t}{4}\right] & \text{if} \quad \left|\frac{x - V\,t}{4}\right| \leq \frac{\pi}{2}, \\ 0 \quad \text{otherwise.} \end{cases}$$

Both the solitons and the compactons are TW solutions, depending, in fact, on a single variable $\xi = x - Vt$, and therefore they are described by ODEs appearing when the TW ansatz $u(t, x) = U(\xi)$ is inserted into the source equations (for details, see e.g. [11]). Thus, it is possible to give a clear geometric interpretation of solitons and compactons. Homoclinic trajectories correspond to both of them in the phase space of the factorized equations. The main difference between the solitary wave and compacton is that the first one is nonzero for any ξ while the second one is nonzero within some compact set. This is because the soliton corresponds to the homoclinic loop usually bi-asymptotic to a simple saddle, and the "time" needed to penetrate the close loop is infinite. The compacton corresponds to the homoclinic loop bi-asymptotic to a topological saddle lying on a singular line. A consequence of this is that the corresponding vector field does not tend to zero when the homoclinic trajectory approaches the stationary point, and the "time" required to penetrate the trajectory remains finite. Let us note that the compacton is in fact a conjunction of the nonzero part corresponding to the homoclinic loop and the trivial constant solution represented by the saddle point [11].

To put it briefly, we maintain the notation traditionally used in more specific sense. Thus, soliton is usually associated with the localized invariant solution to any completely integrable PDE, possessing a number of unusual features [9]. Some of these features are also inherited by the compactons [10, 12, 13]. Within this study solitons are identified with those solutions to (1), which manifest similar geometric features as the "true" wave patterns known under these names, however, it does not mean that they inherit all features of their famous precursors.

The structure of the paper is as follows. In Section 2 we present a dynamical system describing the TW solutions to (1) and find the conditions that contribute to the homoclinic loop appearance. At the end of Section 2 we get the results

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