ADDENDUM TO "MULTIPARTITE STATES UNDER LOCAL UNITARY TRANSFORMATIONS"

SERGIO ALBEVERIO*

Institut für Angewandte Mathematik, Universität Bonn, Wegelerstr. 6, D-53115 Bonn, Germany
(e-mail: albeverio@uni-bonn.de)

LAURA CATTANEO

Institut für Angewandte Mathematik, Universität Bonn, Wegelerstr. 6, D-53115 Bonn, Germany
(e-mail: cattaneo@wiener.iam.uni-bonn.de)

SHAO-MING FEI

Institut für Angewandte Mathematik, Universität Bonn, Wegelerstr. 6, D-53115 Bonn, Germany
Department of Mathematics, Capital Normal University, Beijing 100037, P.R. China
Max Planck Institute for Mathematics in the Sciences, Inselstr. 22, D-04103 Leipzig, Germany
(e-mail: fei@uni-bonn.de)

and

XIAO-HONG WANG

Department of Mathematics, Capital Normal University, Beijing 100037, P.R. China
(e-mail: wangxh@mail.cnu.edu.cn)

(Received September 9, 2005 — Revised May 24, 2006)

In previous work the authors introduced the notion of generic states and obtained criteria for local equivalence of them. Here they introduce the concept of CHG states maintaining the criteria of local equivalence. This fact allows the authors to halve the number of invariants necessary to characterize the equivalence classes under local unitary transformations for the set of tripartite states whose partial trace with respect to one of the subsystems belongs to the class of CHG mixed states.

Keywords: tripartite quantum states, local unitary transformations, entanglement, invariants.

In the paper [1] we exploited the equivalence criterion for the members of a class of bipartite mixed states constructed in [2, 3] to write an equivalence criterion for the members of a class of pure tripartite states. In this Addendum we show how the conditions assumed in [2, 3, 1] can be relaxed.

We first consider (mixed) states on a bipartite system $\mathcal{H}_A \otimes \mathcal{H}_B$, where $\mathcal{H}_A$ and $\mathcal{H}_B$ are finite-dimensional Hilbert spaces of dimension N_A and N_B, respectively. Let ρ be a

*SFB 611; IZKS; BiBoS; CERFIM (Locarno); Acc. Arch. USI (Mendrisio) density matrix defined on $\mathcal{H}_A \otimes \mathcal{H}_B$ with $\mathrm{rank}(\rho) = n \leq N^2$, where $N = \min\{N_A, N_B\}$. ρ can be decomposed according to its eigenvalues and eigenvectors,

$$\rho = \sum_{i=1}^{n} \lambda_i |\varphi_i\rangle\langle\varphi_i|,$$

where λ_i, resp. $|\varphi_i\rangle$, $i = 1, \ldots, n$, are the nonzero eigenvalues, resp. eigenvectors, of the density matrix ρ. $|\varphi_i\rangle$ has the form

$$|\varphi_i\rangle = \sum_{k=1}^{N_A} \sum_{l=1}^{N_B} a_{kl}^i |e_k\rangle \otimes |f_l\rangle, \qquad a_{kl}^i \in \mathbb{C}, \qquad \sum_{k=1}^{N_A} \sum_{l=1}^{N_B} a_{kl}^i a_{kl}^{i*} = 1, \qquad i = 1, \ldots, n,$$

where $\{|e_i\rangle\}_{i=1}^{N_A}$ and $\{|f_i\rangle\}_{i=1}^{N_B}$ are orthonormal bases in $\mathcal{H}_A$ and $\mathcal{H}_B$, respectively, and * means complex conjugation. Let A_i denote the matrix given by $(A_i)_{kl} = a_{kl}^i$. We introduce $\{\rho_i\}$ and $\{\theta_i\}$:

$$\rho_i = \mathrm{Tr}_B\, |\varphi_i\rangle\langle\varphi_i| = A_i A_i^\dagger, \qquad \theta_i = (\mathrm{Tr}_A\, |\varphi_i\rangle\langle\varphi_i|)^* = A_i^\dagger A_i, \qquad i = 1, \ldots, n, \tag{1}$$

with † denoting adjoint. Tr_A and Tr_B stand for the traces over the first and second Hilbert space respectively, and therefore, ρ_i and θ_i can be regarded as reduced density matrices. Let $\Omega(\rho)$ and $\Theta(\rho)$ be two "metric tensor" matrices, with entries given by

$$\Omega(\rho)_{ij} = \mathrm{Tr}(\rho_i \rho_j), \qquad \Theta(\rho)_{ij} = \mathrm{Tr}(\theta_i \theta_j), \qquad \text{for } i, j = 1, \ldots, n, \tag{2}$$

and

$$\Omega(\rho)_{ij} = \Theta(\rho)_{ij} = 0, \qquad \text{for } N^2 \geq i, j > n.$$

In [3] a mixed state ρ is called *generic* if the corresponding "metric tensor" matrices Ω and Θ satisfy

$$\det(\Omega(\rho)) \neq 0 \quad \text{and} \quad \det(\Theta(\rho)) \neq 0. \tag{3}$$

We shall say here that a mixed state ρ is *commuting high generic* or *CHG* if

$$\det(\Omega(\rho)) \neq 0 \quad \textit{or} \quad \det(\Theta(\rho)) \neq 0 \tag{4}$$

and

$$[\rho_i, \rho_j] = [\theta_i, \theta_j] = 0 \tag{5}$$

with ρ_i a full rank matrix. Condition (5) ensures that $A_i A_i^\dagger$ and $A_i^\dagger A_i$ have common eigenvectors.

Similarly we also introduce trilinear expressions $X(\rho)$ and $Y(\rho)$ as

$$X(\rho)_{ijk} = \mathrm{Tr}(\rho_i \rho_j \rho_k), \qquad Y(\rho)_{ijk} = \mathrm{Tr}(\theta_i \theta_j \theta_k), \qquad i, j, k = 1, \ldots, n. \tag{6}$$

PROPOSITION 1. *Two CHG density matrices with nondegenerate Ω (or nondegenerate Θ) are equivalent under local unitary transformations if and only if there exists an ordering of the corresponding eigenstates such that the following invariants have the same values for both density matrices:*

$$\begin{aligned} &J^s(\rho) = \mathrm{Tr}_B(\mathrm{Tr}_A\, \rho^s), \qquad s = 1, \ldots, n, \\ &\Omega(\rho),\ X(\rho) \quad (\textit{or } \Theta(\rho),\ Y(\rho)). \end{aligned} \tag{7}$$