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Heavy particles in incompressible flows: The large Stokes number asymptotics

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Abstract

The dynamics of very heavy particles suspended in incompressible flows is studied in the asymptotics in which their response time is much larger than any characteristic time of fluid motion. In this limit of very large Stokes numbers, particles behave as if suspended in a δ-correlated-in-time Gaussian flow. At those spatial scales where the fluid velocity field is smooth, following Piterbarg [L.I. Piterbarg, The top Lyapunov exponent for stochastic flow modeling the upper ocean turbulence, SIAM J. Appl. Math. 62 (2002) 777] and Mehlig et al. [B. Mehlig, M. Wilkinson, K. Duncan, T. Weber, M. Ljunggren, Aggregation of inertial particles in random flows, Phys. Rev. E 72 (2005) 051104], the two-particle dynamics is reduced to a nonlinear system of three stochastic differential equations with additive noise. This model is used to single out the mechanisms leading to the preferential concentration of particles. Scaling arguments are used to predict the large Stokes number behavior of the distribution of the stretching rate and of the probability distribution function of the longitudinal velocity difference between two particles. As for the fractal character of the particle distribution, strong numerical evidence is obtained in favor of saturation of the correlation dimension to the space dimension at large Stokes numbers. Numerical results at finite Stokes number values reveal that this model catches some important qualitative features of particle clustering observed in more realistic flows.

Keywords: Stochastic flows; Inertial particles; Lyapunov exponents; Preferential concentration

1. Introduction

Dust, impurities, droplets, air bubbles, and other finite-size particles transported by incompressible flows are commonly encountered in many natural phenomena and industrial processes. A salient feature of such suspensions is the presence of strong inhomogeneities in the spatial distribution of particles. This phenomenon is dubbed 'preferential concentration' (see, e.g., [1]). Such inhomogeneities affect the probability to find particles close to each other, and thus influence their possibility to collide or to interact biologically, chemically, or gravitationally. Examples showing the importance of the phenomenon are rain initiation by droplet coalescence in warm clouds [2] or planet formation by dust accretion in the solar system [3]. Engineering applications encompass optimization of spray combustion in diesel engines [4] and in rocket propellers [5].

Particles with a finite size and a mass density different from that of the carrier fluid have inertia. They do not evolve as simple point-like fluid tracers and are termed 'inertial particles'. It can be shown that if their size is below the smallest active scale of the flow (e.g. the Kolmogorov length scale in turbulent flows), the particles are subject to drag, buoyancy, added mass, etc. (see, e.g., [6]). Here we are interested in the limit where particles are not only very small, but also much denser than the surrounding fluid. They then interact with the fluid only through a Stokes viscous drag whose characteristic time (made dimensionless by normalizing it with the typical time scale of the carrier flow) is referred to as the 'Stokes number' *St*.

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Experiments [1,7] and numerics [8–10] show that the degree of inhomogeneity in the spatial distribution of the suspended particles is a non-trivial function of the Stokes number with a maximum at $St \approx 1$.

Quantifying analytically this dependence is an open question. Tools of dissipative dynamical systems are of great use for the investigation of those spatial scales on which the carrier flow is smooth. Indeed, in contrast to tracers in incompressible fluids, inertial particles dynamics is dissipative due to their friction with the fluid. In the position–velocity phase space, their trajectories converge to a dynamically evolving attracting set which is generically multifractal [11,12]. The particle spatial distribution, obtained by projecting this singular set onto the physical space, can also be multifractal [13]. Many observables introduced in the framework of dynamical systems, such as correlation dimension, Lyapunov exponents, or stretching rates, bring important information on particle concentration. Little is known about the dependence of these observables on the Stokes number. Several attempts in determining it have been made in simplified settings: small Stokes number asymptotics [14,15], Gaussian flows with finite [16–18,13] and zero correlation time [19,24,20–23].

In this paper we focus on inertial particles in the limit of very large Stokes numbers. In Section 2 we show that in this limit, no matter the actual nature of the underlying carrier flow provided it is statistically homogeneous and isotropic, the particles do behave as if suspended in a time-uncorrelated Gaussian flow. This result was derived independently in [25]. In Section 3, the approach of [19,21] is applied to write the relative motion of two suspended particles as a three-dimensional (random) dynamical system. This reduced dynamics is related to different observables quantifying inhomogeneities in the particle distribution. Some heuristic understanding of this model is provided.

In Section 4 we extend the scaling arguments developed in [23] to the large Stokes number behavior of velocity differences and of the stretching rate. Predictions are confirmed by numerical experiments which reveal algebraic tails with exponent -3 for the probability distribution function (pdf) of the longitudinal velocity difference between particles. A heuristic argument explaining this behavior is provided. The fractal (correlation) dimension is then investigated numerically in Section 5. Evidence is given that it saturates to the space dimension at sufficiently large values of the Stokes number.

Beside the physical relevance in the large St asymptotics, spatially smooth Gaussian carrier flows without time correlations are valuable models for systematic investigations. We thus study in Section 6 small and intermediate values of the Stokes number. In contrast to the quadratic behavior observed in more realistic flows [15,13,17], it is observed that for $St \ll 1$ the deviation from a uniform distribution is linear in St. However, the general qualitative picture is nonetheless in accordance with observations in real flows. In particular, simulations show that deviations from uniformity are strongest at intermediate values of the Stokes number. As a δ-correlated flow has no structure, this observation questions the phenomenological explanation of particle clustering often found in the literature (see, e.g., [1]). Section 7 is devoted to concluding remarks and summarizes the main findings. Appendix A provides some details on the numerical methods.

2. Model dynamics at large Stokes numbers

For suspensions that are so dilute that collisions, hydrodynamic interactions between particles and retro-action of the particles on the flow can be disregarded, the equations governing the evolution of a spherical particle with density ρ different from that of the carrier fluid ρ_f have been derived in [6]. It was assumed there that the particle radius a is much smaller than the Kolmogorov scale η and that the particle Reynolds number is very small. This implies that the flow surrounding the particle can be approximated by a pure Stokes flow.

In the present paper, we consider impurities that are much heavier than the carrier fluid ($\rho \gg \rho_f$) in the absence of gravity. The time evolution of the particle position $\boldsymbol{X}(t)$ then takes the simple form:

$$\frac{\mathrm{d}^2\boldsymbol{X}}{\mathrm{d}t^2} = -\frac{1}{\tau}\left[\frac{\mathrm{d}\boldsymbol{X}}{\mathrm{d}t} - \boldsymbol{u}(\boldsymbol{X}(t), t)\right], \tag{1}$$

where $\tau = (2a^2\rho)/(9\nu\rho_f)$ is the particle response time, the so-called 'Stokes time', and ν denotes the kinematic viscosity of the carrier fluid.

We are interested in particles with substantial inertia, meaning that $\tau \gg \tau_f$, where τ_f denotes the largest characteristic time of the carrier flow. In a first approximation, such particles relax so slowly to the fluid flow that, along their paths, the local structure of the fluid velocity field changes several times in the interval of time τ. Thus, on the typical time scales of particle motion, the effective fluid velocity field behaves as a time-uncorrelated process. This can be shown formally by rescaling the time as $s = t/\tau$, so that Eq. (1) becomes

$$\frac{\mathrm{d}^2\boldsymbol{X}}{\mathrm{d}s^2} = -\frac{\mathrm{d}\boldsymbol{X}}{\mathrm{d}s} + \tau\boldsymbol{u}(\boldsymbol{X}(\tau s), \tau s). \tag{2}$$

Now, the correlation time of the velocity field being finite and smaller than τ_f by definition of the latter, the central-limit theorem yields $\tau^{1/2}u_i(\boldsymbol{x}, \tau s) \overset{\text{law}}{\sim} \tilde{u}_i(\boldsymbol{x}, s)$ when $\tau \gg \tau_f$, where $\tilde{\boldsymbol{u}}$ is a δ-correlated Gaussian process and where the relation $\overset{\text{law}}{\sim}$ designates equivalence in probability law. With this expression and with transforming s back to the physical time t, (2) yields:

$$\frac{\mathrm{d}^2\boldsymbol{X}}{\mathrm{d}t^2} = -\frac{1}{\tau}\left[\frac{\mathrm{d}\boldsymbol{X}}{\mathrm{d}t} - \tilde{\boldsymbol{u}}(\boldsymbol{X}(t), t)\right]. \tag{3}$$

Hence, particles with very large inertia behave as if suspended in a Gaussian, δ-correlated in time carrier velocity field. For the sake of notation simplicity, we shall omit the tilde on the fluid velocity and refer to $\boldsymbol{u}$ as a δ-correlated carrier flow.

In many real flows, the small-scale properties can be understood by considering a spatially smooth, statistically homogeneous and isotropic velocity field. These spatial properties carry over to the limiting (Gaussian) process, whose