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Multifractal analyses of music sequences

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Abstract

Multifractal analysis is applied to study the fractal property of music. In this paper, a method is proposed to transform both the melody and rhythm of a music piece into individual sets of distributed points along a one-dimensional line. The structure of the musical composition is thus manifested and characterized by the local clustering pattern of these sequences of points. Specifically, the local Hölder exponent and the multifractal spectrum are calculated for the transformed music sequences according to the multifractal formalism. The observed fluctuations of the Hölder exponent along the music sequences confirm the non-uniformity feature in the structures of melodic and rhythmic motions of music. Our present result suggests that the shape and opening width of the multifractal spectrum plot can be used to distinguish different styles of music. In addition, a characteristic curve is constructed by mapping the point sequences converted from the melody and rhythm of a musical work into a two-dimensional graph. Each different pieces of music has its own unique characteristic curve. This characteristic curve, which also exhibits a fractal trait, unveils the intrinsic structure of music.

Keywords: Music; Fractal; Multifractal analysis; Multifractal spectrum; Hölder exponent

1. Introduction

Nature is full of irregular patterns and complicated phenomena. Despite their complicated appearances, ‘self similarity’, that is, the similarity between the whole and a small portion of a system, can be observed in many configurations and phenomena upon closer investigation. Geometry with such scale-invariant features has now been categorized and designated as ‘fractal’ in literature [1]. Many geometries existing in nature are fractal, e.g., a mountain’s profile and the shape of snowflakes. Music, whose origin may be attributed to imitating the harmony of nature’s sound, also demonstrates a fractal property like many other naturally occurring fluctuations do.

Music can be used to express human feelings and emotions toward nature. A few musical notes can be aligned by a composer’s will into a beautiful and pleasant song; whereas the same notes can be arranged into an annoying or discordant noise if randomly aligned. So what is the mystique of music? This is an issue that has been investigated for hundreds of years, but has not been concluded so far. Fractal theory [1], developed in the 1970s, provides an innovative tool for the analysis of a sequence of symbols. By applying fractal tools in the study of music, researchers, including Voss and Hsu, were surprised to discover that the self-similarity property, which is ubiquitous in nature, also exists in music. Such an observation may be regarded as the first step toward a further understanding of what music is and explaining how music simulates the harmony of nature.

1.1. Frequency ratio between music tones

When comparing two tones, a frequency ratio of small number integers (e.g. 1:2 (an octave), 2:3 (a fifth), etc., under the circumstance of ‘just intonation’) indicates a more harmonious sound than a ratio of larger number integers (e.g. 5:6 (a minor third), 15:16 (a minor second), etc.). Just intonation is a system of tuning in which all of the intervals can be represented by ratios of whole numbers, with a strongly-implied preference for the smallest numbers compatible with a given musical purpose. Unfortunately this definition, while

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accurate, does not convey much to those who are not already familiar with the art and science of tuning. The piano and almost all modern keyboard instruments follow the twelve-tone scale; i.e. an octave with a frequency ratio of 1:2 is divided geometrically by even intervals into 12 semitones, each corresponding to one of the seven white or five black keys on the piano, and their frequency and fundamental frequency f_0 satisfy the exponential function of $f_j/f_0 = 2^{j/12}$. The twelve-tone scale differs from just intonation in frequency ratio; e.g. a perfect fourth consists of 5 semitones with a frequency ratio of $2^{5/12} = 1.3348$, which is close to 4/3; a perfect fifth consists of 7 semitones with a frequency ratio of $2^{7/12} = 1.4983$, which is nearly 3/2. Both are ratios of smaller integers. A diminished fifth, however, has 6 semitones with a frequency ratio of $2^{6/12} = 1.4142$, which is almost 1000/707. This is not a ratio of small integers. Therefore, such an interval has been traditionally considered dissonant and is rarely used in classical pieces.

1.2. Music as $1/f$ noise

Before discussing the relationship between music and fractal theory, let us focus on a particular type of noise — $1/f$ noise first. Mandelbrot proposed that there is a kind of sound in which the quality is unaffected by changes in play speed, and called this sound 'scaling noise' [1]. The plainest example of scaling noise is 'white noise'. Suppose a time series is produced in accordance with temporal variations of white noise, a calculation of its power spectral density $S(f)$ reveals that the relationship between $S(f)$ and f can be stated as $S(f) \propto f^{-\beta}$, where scaling exponent $\beta = 0$, indicating its monotonousness at whatever play speed. In other words, white noise is a mixture of frequency components from a wide range that are randomly and completely combined; its features are utmost randomness and totally unrelated points. Brownian noise is another type of scaling noise with scaling exponent $\beta = 2$. It depicts Brownian movement or random walk, with the strongest correlation among points within a characteristic time scale.

On the other hand, after conducting a spectral analysis on various types of music, including classical music (Bach, Mozart, Beethoven …) and modern jazz, Voss and Clarke [2,3] discovered that musical works of various melodies and styles share a similar tendency toward a $1/f$ spectrum. In fact, music featuring a $1/f$ spectrum happens to be a $1/f$ noise intermediary between the flat spectrum of white noise and the steep $1/f^2$ spectrum of Brownian noise. It is a kind of scaling noise, too. However, neither white noise nor Brownian noise can be called music; the former is so random and unassociated that it becomes uninteresting, while the latter has over-emphasized connections and lacks charm. Only $1/f$ noise can merge the randomness and orderliness into a naturally pleasant and attractive whole [4,5].

1.3. Fractal geometry in music

Observation of time series of $1/f$ noise with various time scales reveals statistical self-similarity. That is to say, any enlargement or reduction of the timeline would not affect the tendency of fluctuation. Mandelbrot called such behavior scale invariance. Furthermore, $1/f$ noise features a long-range correlation, or retaining memory over a rather long period of time. Coincidently, nature is saturated with the $1/f$ phenomenon, as seen in a mountain contour and the fluctuation of a river's water level, whose variations also have the traits of scale invariance and long-range correlation. The spectral analysis in the study by Voss and Clarke substantiated the assumption that music imitates characteristics of temporal variations demonstrated by nature and the universe, and that music features fractal geometry.

As mentioned above, Voss and Clarke, from their analysis on the power spectrum $S(f)$ of musical signals of various styles, observed fractal distribution approximating to $1/f$ in power spectra of both loudness and frequency fluctuation (waves of melody). However, they also pointed out that such a phenomenon is not found in all ranges of frequency; instead, it is only so between 100 Hz and 10 kHz. In cases of high frequency (100 Hz–2 kHz), $S(f)$ is not molded as $1/f$. Hence Voss and Clarke suggested that, within a certain range, signal fluctuations of most musical works feature long range correlation, and the exponents of the power spectrum may also be associated with fractal content of music.

In the 1990s, Hsu and Hsu [6] discovered from analysis of music scores by Bach and Mozart that, in general, the difference in pitch j between two successive notes (i.e. the melody) and the frequency of their appearance F have an exponential relation, which can be stated as $F \propto j^{-D}$, where D is dimension. Values of the exponent D in various musical scores range between 1 and 3, but they are not integers. As the dimension is not a whole number, the frequency of pitch variation in music can be categorized as fractal geometry. In order to visualize music, Hsu and Hsu [7] used the j value to represent each musical note in a score, marked them in order of appearance on coordinate axes (x, y), forming a curve, and then diminished the sequence length by labeling points at intervals of 2, 4, and 8 … points. The reduced curve looked much the same as the original one, and the style remained unaffected. Therefore, musical scores share the feature of self-similarity with fractal geometry [8].

In addition, in a recent study Shi [9] employed the calculation method of the Hurst exponent to examine the pitch sequence fashioned in folk songs and piano pieces. Their results indicated that music sequences have the property of long range correlation and the fundamental principle of music is the balance between repetition and contrast. Further, Bigerelle and Iost [10] applied the 'Variance Method' to study the fractal dimensions in 180 musical works of various styles. Based on statistical results, they proposed that various music pieces could be categorized by fractal dimension. Madison [11] used a similar approach to study different musical scores with Hurst exponents, which were found thereafter to play an important role in the emotional expression of musical performance. The study by Manaris et al. [12] of a 220-piece corpus (baroque, classical, romantic, 12-tone, jazz, rock, DNA strings, and random music) revealed that esthetically pleasing music might be describable under the Zipf–Mandelbrot law. Gunduz and Gunduz [13] studied the mathematical structures of six songs