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Efficient computation of capillary–gravity generalised solitary waves

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HIGHLIGHTS

- A new formulation for steady capillary–gravity waves is presented.
- A numerical algorithm to solve the Babenko equation is described in detail.
- The performance of this algorithm is illustrated on various examples of generalised solitary waves.
- Internal flow structure for these waves is shown.
- The code is freely available in open source.

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ABSTRACT

This paper is devoted to the computation of capillary–gravity solitary waves of the irrotational incompressible Euler equations with free surface. The numerical study is a continuation of a previous work in several points: an alternative formulation of the Babenko-type equation for the wave profiles, a detailed description of both the numerical resolution and the analysis of the internal flow structure under a solitary wave. The numerical code used in this study is provided in open source for those interested readers.

1. Introduction

The formation and dynamics of capillary–gravity waves on the surface of incompressible fluids constitute a research topic of permanent interest from experimental, theoretical and computational point of view [1–4]. These solutions have been also studied using analytical asymptotic methods in various model equations (such as KdV5), see [5–7] and also the contributions to the edited volume [8]. This paper presents in detail an efficient numerical algorithm for computing various steady capillary–gravity solitary waves for the irrotational Euler equations with a free surface. In particular, this algorithm can be used to compute generalised multi-hump solitary waves [9]. In addition to the detailed algorithm, the present paper provides some physical characteristics of the generalised solitary waves that are not described in [9]. The highlights can be assembled in three points.

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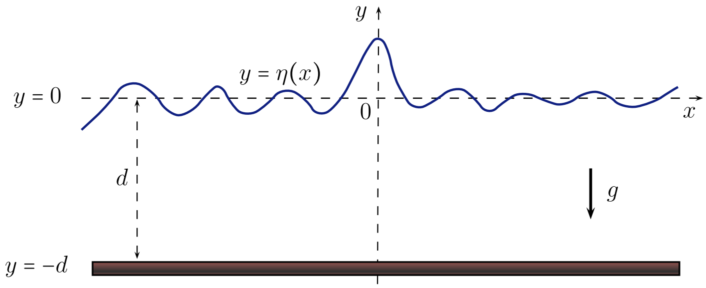

Fig. 1. Definition sketch of the physical domain.

The first one concerns the mathematical formulation of the equations for the wave profiles. Different from other approaches used in the literature to reformulate the Euler system on the free surface, [3,10,11], the one employed in [9] is based on the conformal mapping technique [12], in order to transform the system into an integro-differential equation of Babenko type [13]. (This was previously developed for gravity solitary waves in [14–16].) As mentioned in [9], one of the advantages of this formulation is the preservation of the type of nonlinearity by the transformation [17]; it has implications on the convergence of the numerical method. As in [9], the numerical results on steady capillary–gravity waves presented here are obtained with a pseudo-spectral discretisation of the corresponding periodic problem and the application of the Levenberg–Marquardt (LM) algorithm, [18–20], to the resulting discrete systems for the Fourier components of the approximation.

A second highlight of the present paper is then a detailed description of the numerical procedure and its implementation to the model formulation. The performance of the method is shown through several numerical experiments computing different travelling waves.

Finally, the emergence of generalised solitary waves shown in [9], under gradually increasing values of the Bond number is used in a third highlighted point of the present study, where some properties of various physical fields under these waves are emphasised.

The manuscript is organised as follows. In Section 2, we summarise the application of the conformal mapping technique to the original Euler equations, explained in [9] in more detail. Section 3 is devoted to a detailed description of the numerical method while the main numerical experiments are presented in Section 4. Finally, the main conclusions and perspectives are outlined in Section 5. With the aim of involving a wider audience, the numerical code used in computations below is available to download as open source [21]. Thus, the claims made in this study can be easily verified by interested readers.

2. Mathematical model

Our starting point is the mathematical model for a potential flow induced by a solitary wave described in [9], which is summarised here. The physical assumptions involve an inviscid, homogeneous fluid in a horizontal channel of constant depth; the channel is modelled, above, by an impermeable free surface where the pressure is equal to the surface tension due to capillary forces, and bounded below by a fixed impermeable horizontal seabed.

The mathematical description in Cartesian coordinates moving with the wave (with x as the horizontal coordinate and y the upward vertical one) is sketched in Fig. 1: d stands for the mean depth of the channel, the bottom is posed at $y = -d$ while $y = \eta(x)$ denotes the free surface elevation from the mean water level at $y = 0$. The zero mean level condition of the free surface is redefined in order to cover the computations of generalised solitary waves (see [9] for details).

Let ϕ and ψ be the velocity potential and the stream function, respectively, and consider the complex potential $f \equiv \phi + \mathrm{i}\psi$. They define a conformal mapping

$$z \;\mapsto\; \zeta \;\equiv\; (\mathrm{i}\psi_{\mathrm{s}} - f)/c, \tag{2.1}$$

where ψ_{s} and ψ_{b} are the constant traces of ψ at the upper and lower boundaries, respectively, with $-c$ standing for the mean flow velocity and satisfying

$$c \;\equiv\; -\left\langle \frac{1}{d}\int_{-d}^{\eta} u(x,y)\,\mathrm{d}y \right\rangle = \frac{\psi_{\mathrm{b}} - \psi_{\mathrm{s}}}{d}. \tag{2.2}$$

The conformal mapping (2.1) transforms the fluid domain into the strip $-\infty \leqslant \alpha \leqslant \infty$, $-d \leqslant \beta \leqslant 0$, where $\alpha \equiv \mathrm{Re}(\zeta)$ and $\beta \equiv \mathrm{Im}(\zeta)$. On this domain, the free surface η can be modelled by a nonlocal Babenko-type equation of the form [9]

$$\mathscr{C}\left\{ B\,\eta - \frac{g\,\eta^2}{2} + \tau - \frac{\tau\,(1+\mathscr{C}\{\eta\})}{\sqrt{(1+\mathscr{C}\{\eta\})^2 + \eta_\alpha^2}} \right\} = g\,\eta\,(1+\mathscr{C}\{\eta\}) - \frac{\mathrm{d}}{\mathrm{d}\alpha}\left\{ \frac{\tau\,\eta_\alpha}{\sqrt{(1+\mathscr{C}\{\eta\})^2 + \eta_\alpha^2}} \right\} + K. \tag{2.3}$$