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A note on incremental equations for a new class of constitutive relations for elastic bodies

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HIGHLIGHTS

- Incremental equations are obtained for some new constitutive relations.
- The propagation of small waves is studied for an infinite solid.
- The influence of the initial time independent stress distribution is considered.

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ABSTRACT

Some new classes of constitutive relations for elastic bodies have been proposed in the literature, wherein the stresses and strains are obtained from implicit constitutive relations. A special case of the above relations corresponds to a class of constitutive equations where the linearized strain tensor is given as a nonlinear function of the stresses. For such constitutive equations we consider the problem of decomposing the stresses into two parts: one corresponds to a time-independent solution of the boundary value problem, plus a small (in comparison with the above) time-dependent stress tensor. The effect of this initial time-independent stress in the propagation of a small wave motion is studied for an infinite medium.

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1. Introduction

In the recent years some new constitutive relations have been proposed for the modelling of elastic bodies [1–9]. One of such new relations corresponds to the implicit equation $\mathfrak{F}(\mathbf{B}, \mathbf{T}) = \mathbf{0}$, where $\mathbf{B}$ is the left Cauchy–Green deformation tensor and $\mathbf{T}$ is the Cauchy stress tensor [1,2]. In the particular case the gradient of the displacement field $\mathbf{u}$ would be very small $|\nabla \mathbf{u}| \sim O(\delta)$, $\delta \ll 1$, we have the approximation $\mathbf{B} \approx 2\boldsymbol{\varepsilon} + \mathbf{I}$ (where $\boldsymbol{\varepsilon}$ is the linearized strain tensor), and it has been proved that from such implicit relation we obtain a constitutive equation of the form [6]

$$\boldsymbol{\varepsilon} = \mathfrak{f}(\mathbf{T}). \tag{1}$$

This special case obtained from the general implicit relation is very important on its own, since it can be used to study problems, where we have bodies that behave elastically and nonlinearly, but where the strains are small [10–13,6]. Different applications of such constitutive equation can be found in fracture mechanics [14,11,15], in the modelling of some metals [16,17], and in rock mechanics [18] (if in a first approximation we consider rock as an elastic material).

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In this note we study the wave propagation phenomena for an elastic body considering (1), the interested reader can see [9,12] for some other results concerning wave propagations for (1), and [19] where some results are shown for an extension of (1) to the case of considering viscoelastic behaviour. In the present work we assume that the body is under the influence of a stationary stress field $\mathbf{T}_o = \mathbf{T}_o(\mathbf{x})$, to which we add a small time dependent stress field $\Delta\mathbf{T}(\mathbf{x}, t)$, where $\|\Delta\mathbf{T}\| \ll \|\mathbf{T}_o\|$. Incremental equations are obtained, and the problem of small stress wave propagation is considered. In this work we restrict ourselves to plane problems for materials defined by (1). Even though strains are assumed to be small the stress–strain relation (1) may be nonlinear. We start by producing the equations describing small stresses superimposed upon large for this case when the strain is written as a function of the stress. By imposing conditions that guarantee the invertibility of the incremental stress–strain relation, and assuming the small motion to be that of a plane wave, we arrive at the secular equation for the problem at hand. The requirement of a non-trivial amplitude for the wave, results in a set of conditions for the constants that appear in the stress–strain relation. Using some ideas from the theory of quadratic forms we find some inequalities that these constants should satisfy in order for the material to admit waves with real velocity. It is worth stressing that in this way the acoustic tensor of the material is produced as well.

In Section 2 some basic equations are presented, in particular a summary of the new constitutive relations mentioned previously. In Section 3 the incremental equations are derived for these constitutive relations, while in Section 4 the particular case of plane strains is considered. In Section 5 the problem of wave propagation in an infinite medium is studied. Finally, in Section 6 some final remarks and comments are given.

This work is partially based on the results presented in the Master degree thesis by Arrue [20].

2. Basic equations

2.1. Kinematics

Let X denote a particle of a body $\mathscr{B}$, where the position of this particle in the unstressed reference configuration is $\mathbf{X} = \kappa_r(X)$, where the reference configuration is $\kappa_r(\mathscr{B})$. We assume there exists a one-to-one mapping $\boldsymbol{\chi}$ such that the position of the particle X in the current deformed configuration $\mathbf{x}$ is given as $\mathbf{x} = \boldsymbol{\chi}(\mathbf{X}, t)$, where the current configuration is denoted $\kappa_t(\mathscr{B})$.

The deformation gradient, the left Cauchy–Green strain tensor and the linearized strain tensor are defined, respectively as:

$$\mathbf{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}}, \qquad \mathbf{B} = \mathbf{F}\mathbf{F}^{\mathrm{T}}, \qquad \boldsymbol{\varepsilon} = \frac{1}{2}(\nabla\mathbf{u} + \nabla\mathbf{u}^{\mathrm{T}}), \tag{2}$$

where the displacement field $\mathbf{u}$ is given as $\mathbf{u} = \mathbf{x} - \mathbf{X}$ and $J = \det\mathbf{F} > 0$.

2.2. Equation of motion and constitutive relations

The equation of motion is

$$\operatorname{div}\mathbf{T} + \rho\mathbf{b} = \rho\ddot{\mathbf{u}}, \tag{3}$$

where $\mathbf{T}$ is the Cauchy stress tensor, ρ is the density of the body, $\mathbf{b}$ represents the body force and $\ddot{\mathbf{u}} = \frac{\partial^2\mathbf{u}}{\partial t^2}$. More details about the kinematics of deforming bodies and about the equation of motion can be found, for example, in [21].

One of the implicit constitutive relations proposed by Rajagopal [1,2] corresponds to

$$\mathfrak{F}(\mathbf{B}, \mathbf{T}) = \mathbf{0}. \tag{4}$$

It has been shown in [13, see Section 2.2 therein] that if $|\nabla\mathbf{u}| \sim O(\delta)$, $\delta \ll 1$, then considering the approximation $\mathbf{B} \approx 2\boldsymbol{\varepsilon}+\mathbf{I}$, from (4) we obtain the constitutive relation

$$\boldsymbol{\varepsilon} = \mathfrak{f}(\mathbf{T}). \tag{5}$$

As explained in the Introduction this constitutive relation can be used to study problems, where we have bodies that behave in a nonlinear manner, but where the strains are small and the stresses can be arbitrarily 'large' (in comparison with some characteristic value σ_o for the stress).

Regarding (5), let us assume additionally that there exists a scalar function $\Pi = \Pi(\mathbf{T})$ such that (see [7,22]) $\mathfrak{f}(\mathbf{T}) = \frac{\partial \Pi}{\partial \mathbf{T}}$, i.e.:

$$\boldsymbol{\varepsilon} = \frac{\partial \Pi}{\partial \mathbf{T}}. \tag{6}$$

2.3. Summary of the boundary value problem

In order to solve some boundary value problems, we need to find $\mathbf{T}$ and $\mathbf{u}$ that satisfy (3), $(2)_3$ and (5)

$$\operatorname{div}\mathbf{T} + \rho\mathbf{b} = \rho\ddot{\mathbf{u}}, \qquad \frac{1}{2}(\nabla\mathbf{u} + \nabla\mathbf{u}^{\mathrm{T}}) = \boldsymbol{\varepsilon}, \quad \boldsymbol{\varepsilon} = \mathfrak{f}(\mathbf{T}), \tag{7}$$

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