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Scattering of Lamb waves from a discontinuity: An improved analytical approach

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HIGHLIGHTS

- An analytical model (CMEP) was created to predict scattered Lamb waves from a damage.
- CMEP method is illustrated using the benchmark step problem.
- CMEP verified the results from axial–flexural wave model at low frequencies.
- CMEP and FEM results agreed very well, while CMEP being orders of magnitude faster.
- CMEP predicted the scattered wave field over a wide range of frequencies.

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ABSTRACT

This paper presents an efficient and accurate analytical method to calculate the scattering of straight-crested Lamb waves from geometric discontinuities. In this method, the scattered field is expanded in terms of complex Lamb wave modes with unknown amplitudes. These unknown amplitudes are obtained from the boundary conditions using a vector projection utilizing the power expression. The process works by projecting the stress conditions onto the displacement eigen-spaces of complex Lamb wave modes and vice-versa. We call this technique "complex modes expansion with vector projection" (CMEP). Unlike other methods, the CMEP approach is versatile and can be readily applied to notches, cracks, or disbonds. In this paper, the methodology is illustrated by applying the CMEP method to a benchmark problem — the geometric discontinuity created by a step in the thickness of a plate. For method verification, the finite element method (FEM) and the axial–flexural analytical model were used. The FEM analysis was conducted in the frequency domain with non-reflecting boundaries. It was found that the CMEP results correspond very well with the FEM results over a wide frequency–thickness range up to 1.5 MHz-mm. The axial–flexural model was used to verify only the CMEP asymptotic behavior toward zero frequency where frequency-domain FEM encounters difficulties. Our study shows that the computational efficiency of CMEP is orders of magnitude higher than FEM. The paper ends with a discussion of how the CMEP method may be extended to the fast and accurate analysis of realistic damage situations.

1. Introduction

1.1. State of the art

In the field of non-destructive evaluation (NDE) and structural health monitoring (SHM), methods using ultrasonic plate guided waves are popular for their potential use in the inspection of large structures. Due to this popularity, the

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prediction of the scattering of Lamb waves from damage has been a major focus for researchers in NDE and SHM. Damage characterization in particular, as an inverse problem, requires fast and accurate prediction of scattered waves. But the solution of the scattering problem is highly challenging because of the existence of multiple dispersive modes of Lamb waves at any frequency along with mode conversion at the damage location. Therefore, well-developed numerical methods such as the finite element method (FEM) and the boundary element method (BEM) have been popular [1–4]. However commercial FEM codes are time consuming and they do not provide much insight of the wave field in the structure, especially near the damage location. Therefore, for efficiency of simulation, researchers developed hybrid methods using FEM, BEM, and normal mode expansion [2,4–6]. Glushkov et al. proposed the layered element method (LEM) which, unlike BEM, satisfies the plate boundary conditions by formulation [7]. They also proposed a simplified analytical model based on Kirchhoff plate theory for fast simulation [8]. However, the prospect of developing analytical models based on normal mode expansion has also attracted attention for possible speed and accuracy [9–18]. Gunawan et al. [14] developed a semi analytical solution to the reflection of an oblique guided wave from the free edge of a plate by substituting the stress free edge boundary conditions into the complex orthogonality relations. However, this approach was not applied to a geometric discontinuity problem such as the step problem. Feng et al. [16–18] used the normal mode expansion method to analyze thickness discontinuities in a plate under the plane strain condition. They split the initial plate into layers of real and virtual subplates in order to achieve the step discontinuity. Then, free-end reflection conditions were imposed on some of the subplates to generate a system of algebraic equations which yielded the modal expansion coefficient.

One of the main challenges of the scattering problem is to satisfy the thickness dependent boundary conditions at the discontinuity [6,11]. Gregory et al. [10,19] developed the 'projection method' to satisfy these thickness dependent boundary conditions. This method was developed to predict the singularity in stresses in the case of geometric discontinuities [13]. A scalar form of the projection method was also used by Grahn [12]; though simple, this approach proved to be not very stable and to have slow convergence due to the use of simple sine and cosine functions [17]. Moreau et al. [15] used the displacement components of the complex Lamb wave modeshapes instead of simple sine and cosine functions and attained a faster convergence in the projection method.

1.2. The novelty of present paper

In this paper, we present an efficient analytical method for the prediction of Lamb wave scattered field using complex mode expansion and vector projection. We improved in several ways upon previous authors. One improvement is that we modified the projection method to take advantage of the power flow associated with Lamb wave modes. As different from the references [12–15], the stress boundary conditions are projected onto the conjugate of the displacement modeshapes and the displacement boundary conditions are projected onto the conjugate of the stress modeshapes. Therefore this method transforms stress and displacement equations into the power equations. Another improvement is that we applied the projection to all the unknown wave fields. Our approach leads to fast convergence in terms of the number of complex modes needed in the modal expansion because it creates dominant diagonal terms in the matrix equation. It also ensures that, when the method has converged, the power balance is automatically satisfied. We call this method complex modes expansion with vector projection (CMEP). It is a Galerkin type approach where we implemented vector projection of the boundary conditions directly using the power flow expression.

Other approaches exist in the literature using the orthogonality between modal stresses and displacements [16–18] but they are different from our method because they need to assume virtual wave guides and vertical free-ends to implement the orthogonality relations.

In this paper, we demonstrate the CMEP method using the step problem which is one of the simplest forms of plate discontinuities. However, in terms of analytical modeling, the step problem poses the same challenges as a realistic damage. Therefore, for any new method aiming to solve the scattering problem, the step problem can be used as a benchmark problem even though it may lack a direct practical relevance.

The paper is organized as follows: Section 2 describes the method using the step problem; Section 3 verifies the asymptotic behavior of CMEP results at frequencies close to zero by comparing them with the analytical axial–flexural results; Section 4 validates CMEP results by comparing them with the results from finite element models over a large range of frequencies; and finally, Section 5 discusses how the CMEP method can be extended from the step problem illustrated here to more realistic damage cases such as cracks, corrosion, and disbonds.

2. Analytical model for Lamb waves scattering

Lamb waves are plate guided waves with multiple propagating modes at any frequency of excitation. The characteristic equation of Lamb waves is known as the Rayleigh–Lamb Equation [20] and is expressed in terms of the dimensionless wavenumber $K = \xi d$ and dimensionless frequency $\Omega = \omega d / c_S$ as

$$\begin{aligned} f_S(K,\ \Omega) &= \left(K^2 - a^2\right)^2 \sin a \cos b + 4abK^2 \cos a \sin b = 0 \quad \text{(Symmetric)} \\ f_A(K,\ \Omega) &= \left(K^2 - a^2\right)^2 \sin b \cos a + 4abK^2 \cos b \sin a = 0 \quad \text{(Antisymmetric)} \end{aligned} \tag{1}$$