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Effect of initial stress on Love wave propagation at the boundary between a layer and a half-space

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HIGHLIGHTS

- Uses general theory of initial stress in nonlinear elasticity.
- Illustrates the effect of initial stress on the speed of Love waves.
- For tensile initial stress, wave speed increases with increasing wave number.
- For compressional initial stress, wave speed decreases with increasing wave number.

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ABSTRACT

In this paper, a discussion on the effects of initial stress and finite deformation on the speed of Love waves propagating along the boundary between a half-space and a layer is carried out. Here, both the half-space and the layer are assumed to be incompressible and initially stressed in their reference configuration. The initial stress in this problem is not associated with a finite elastic deformation which is in contrast to the situation where the initial stress is a *pre-stress* that is accompanied by a finite deformation. A general formulation of the equations governing incremental motions is provided, and then the specialized case of in-plane motions is considered. The theory of superposition of infinitesimal deformations on finite deformations is applied for initially-stressed incompressible materials. The principal plane of the underlying (homogeneous) deformation with a uniform initial stress is assumed to be coaxial with the finite deformation. With this specialization, the combined effect of initial stress and finite deformation on the speed of Love waves is discussed and the results are analyzed graphically with the main emphasis on the effect of initial stress on wave speed.

1. Introduction

In this paper, a discussion on the propagation of Love-type waves at the boundary between a half-space and a layer is carried out with the assumption that both materials are initially-stressed in their reference configuration. Initial stresses occur in the earth's crust due to various tectonic events, in mechanical parts and structural elements during their manufacture and assembly, in composite materials when they are formed, in blood vessels, in rocks, etc. Thus, the properties of initial stress and its effects on the elastic waves are an important study while taking up problems in the mechanics of materials and composites, geophysics, seismology, rock mechanics, bio-mechanics, nondestructive testing of materials, etc., to name a few. Such an initial stress exists in the materials when there is no accompanying finite deformation since many

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materials are internally stressed in their unloaded reference configuration. The term *initial stress* is used in its broadest sense, irrespective of how the stress develops. This includes situations where the stress is a pre-stress, which exists due to some finite deformation and situations in which the initial stress arises from some other process, such as manufacturing or growth, and is present in the absence of applied loads, in which case it is referred to as *residual stress*, as in the definition adopted by Hoger [1], who later showed the material must be non-homogeneous for the residual stresses to exist [2].

In the classical theory, surface waves with horizontally polarized displacement do not exist in a purely elastic half-space (see, for example, [3]). The idea that shear horizontal waves can exist in a layer was elaborated for the first time by Love [4]. Later, Biot [5] studied the influence of initial stress on elastic waves. The theory developed by Biot in his monograph [6] is widely used in literature but the elastic modulus tensor in this theory does not depend on the initial stress. Earlier studies on this topic include work by Hadamard [7], Brillouin [8] and Love [9]. The study on propagation of Love-type surface waves in an elastic body subject to pre-stress was initiated by Hayes and Rivlin [10], who presented results for compressible and incompressible isotropic elastic materials. This work was followed by that of Flavin [11], who studied surface waves propagating along a principal direction of strain in an isotropic elastic Mooney–Rivlin material and in a general direction for a neo-Hookean material. A discussion on the propagation of shear horizontal waves in a composite medium under initial stresses can be found in [12]. Several contributions were made by Willson [13–16], who examined the properties of surface waves for a variety of incompressible and compressible isotropic elastic materials for different states of pre-stress. Chadwick and Jarvis [17] analyzed surface waves in a pre-stressed compressible isotropic elastic material.

Many authors use the theory developed in [6] where the governing equations include initial stress in the context of linear elasticity. For example, the effect of initial stress on Love waves in a non-homogeneous orthotropic elastic layer was discussed by Abd-Alla and Ahmed [18]. A comprehensive collection of the work done on elastic waves in bodies with initial (residual) stresses can be found in [19] and the reference therein. Various problems related to elastic waves in initially-stressed anisotropic materials are found in [20–23] and, Pal and Ghorai [24]. The behavior of Love waves in a functionally graded material layered non-piezoelectric half-space with initial stress was discussed by Qian et al. [25] whereas the effect of compressive initial stress and gravity field on S-waves was studied by Gupta and Vishwakarma [26]. Singh et al. [27], Gupta et al. [28], Kundu et al. [29] and, Kundu et al. [30] investigated the properties of SH-waves in initially-stressed heterogeneous materials.

A major contribution to the study of initial stress on plane waves is due to Man and Lu [31] who mainly followed Hoger [1] and used the concept of incremental elasticity tensor dependent upon the initial stress tensor. The reader is referred to the discussion in §8 of Man and Lu [31] where is was found that the phase velocity of Love waves is a function of the initial stress. Dowaikh and Ogden [32] studied the infinitesimal surface wave propagation in a pre-stressed incompressible elastic half-space whereas a similar analysis was carried out by Dowaikh [33] for Love-type waves. For the problem formulation in the present paper, the theory of invariants and the expressions of the fourth-order elasticity tensor are used as were developed in [34]. See also the discussion in [34–37] and, Nam [38], where various properties of initial stress and its effect on elastic waves are investigated and the fact that the presence of an initial stress imposes conditions on the form of the strain energy function is emphasized. In contrast to the extensive studies for infinitesimal deformations, the problem to see the effect of residual stress on the elastic behavior of finitely-deformed elastic materials, in particular, cylinders with azimuthal shear and torsion, was carried out by Merodio et al. [39].

In this paper, the primary concern is to study the effect of initial stress on the propagation of Love waves in an incompressible material using the theory of infinitesimal deformation superimposed on a finite deformation. It may be noted that, in the absence of initial stress, the material is assumed to be isotropic. In Section 2, the basic equations and the constitutive law for an initially stressed elastic material in terms of invariants are collected. Section 3 provides the incremental equations of motion and detailed expressions for the elasticity tensor of an initially stressed material. In Section 4, specialized incremental equations governing two-dimensional motions are given in a principal plane of an underlying pure homogeneous deformation which is also a principal plane of a (uniform) initial stress. In Section 5, the in-plane equations of motion are used to analyze the speed of Love waves at the boundary of a half-space that is jammed with a layer of thickness h. Specialized results for a prototype strain energy function with uniform uniaxial initial stress is presented. In particular, the dispersion relations for Love waves is presented and the dependence of the wave speed on the deformation and the initial stress is illustrated through various graphs.

2. Kinematics, equilibrium and invariant formulation of the constitutive law for an elastic solid with initial stress

Consider a material body which is initially-stressed in its reference configuration. Let this reference configuration is denoted $\mathcal{B}_r$, and its boundary by $\partial\mathcal{B}_r$. The initial stress in $\mathcal{B}_r$ is denoted $\boldsymbol{\tau}$ and is not necessarily associated with an elastic deformation. It may be noted that the origin of this stress is not of primary concern in this problem. In the absence of body forces, the initial stress is in equilibrium in $\mathcal{B}_r$ and hence $\text{Div}\,\boldsymbol{\tau} = \mathbf{0}$. For the problem under discussion, without loss of generality, the initial stress $\boldsymbol{\tau}$ is assumed to be *symmetric*, homogeneous and uniform. By contrast, when the initial stresses are residual stresses they are inhomogeneous and dependence on position then needs to be accounted for.

Let the position vector of a material point be $\mathbf{X} \in \mathcal{B}_r$. After the material body undergoes a deformation, let $\mathcal{B}$ denote the new configuration, with boundary $\partial\mathcal{B}$. In $\mathcal{B}$, the material point $\mathbf{X}$ has the new position $\mathbf{x} = \boldsymbol{\chi}(\mathbf{X})$, where the vector function $\boldsymbol{\chi}$ defines the deformation for $\mathbf{X} \in \mathcal{B}_r$ and possesses appropriate mathematical properties. The deformation gradient tensor