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A weak formulation for interior acoustic analysis of enclosures with inclined walls and impedance boundary

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HIGHLIGHTS

- A weak approach is presented to study acoustic properties of irregular enclosures.
- The decomposition technique is adopted for mid-frequency response analysis.
- Effects of predetermined constants on the accuracy of present results are performed.
- Key parametric studies concerning geometries and impedance boundary are investigated.
- The present approach is meaningful for acoustic engineering.

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ABSTRACT

A weak variational principle based approach is presented in this paper to study the sound field inside the acoustic enclosures with walls in arbitrary inclination and impedance conditions. The whole acoustic domain is firstly divided into several sub-cavities with trapezoidal and rectangular faces, and each sub-cavity is coupled with adjacent ones by matching the required continuity constraints on the interfaces on the basis of a modified variational principle and least-squares weighted residual method. By using this domain partitioning strategy, high-order acoustic modes and responses can be easily achieved. Chebyshev orthogonal polynomials of the first kind are employed as the wholly admissible unknown sound pressure functions for each sub-cavity without meshing process like FEM/BEM does, and then each physical domain is mapped into a square spectral domain. To demonstrate the convergence, accuracy and stability of the approach, the modal and sound response analyses of several configurations of cavities are examined and compared with available analytical solutions, or those obtained by using FEM. Effects of the weighted parameters together with the number of truncated polynomial terms and the divided cavity segments on the accuracy of present solutions are investigated. Key parametric studies concerning the influences of the geometrical properties as well as the impedance boundary of enclosing walls are also performed. It is demonstrated that the present method is a computationally efficient way to achieve interior sound predictions in mid-frequency range with a satisfactory accuracy of solutions.

1. Introduction

With the increasing requirements for acoustic performance of products as well as comfort working and living conditions, the study on sound field inside a finite spatial domain has been of increasing interest for both scientific research and practical

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applications. Typical acoustic-related engineering examples include automobiles, underwater vehicles, aircraft and office partitions etc. In such applications, a bounded cavity is a meaningful representation of engineering problems regarding the noise control and acoustic treatment. Hence, an exhaustive understanding of the acoustic characteristics of enclosure is imperative for satisfactory acoustic design with more restrictive legal regulations with respect to noise emission levels. The initial work in interior sound field simulation can be traced back to 1950's [1,2]. Since then, a large amount of effort has been devoted to investigating the acoustic behavior of enclosure cavity. The sound field analysis can be easily made for the cavities with simple geometry and perfectly rigid walls. However, cavity with arbitrary wall impedances is of common interest and important for real life such as the use of porous materials in room wall. As a consequence, this class of problem is still challenging owing to the highly nonlinear and transcendent nature of the acoustic eigenvalue equation. In order to resolve this problem, numerical root seeking algorithms are needed, such as Newton's method [3], homotopic continuation procedure [3] and Newton/generalized bisection method [4]. In addition to effective root researching algorithms, some analytical and numerical methods are also developed for modeling pure acoustic cavities, such as the finite element method (FEM) [5–8], boundary element method (BEM) [6,9–12], equivalent source technique [13], Green's function formulation [14], Rayleigh–Ritz method [15,16], etc.

In general, the analysis of regular-shaped cavity is easy to deal with using modal-based theoretical framework, which is useful in revealing the major sound behavior. However, many realistic enclosures are not suitable for being approximated to simple and regular forms, making analytic solutions impossible. Therefore, the general formulation for predicting acoustic properties of irregular-shaped cavity is crucial to the acoustic analysis and desired design of cabins (cars, airplanes, etc.). Bouillard et al. [17] developed an improved element-free Galerkin method for the acoustic problem of three-dimensional complex geometries. Missaoui and Cheng [18], Li and Cheng [19,20] used the combined integro-modal (CIM) approach to investigate the possible changes in acoustic characteristics of the rectangular-like cavity due to the geometric distortion. Using the similar method, Sum and Pan [21] studied the sound decay time in a trapezoidal cavity in terms of the effects of the inclined wall and impedance surface location. Their studies [19–21] considered a rectangular-like enclosure with one inclined wall. Recently, Desmet et al. proposed the wave based method (WBM) based on an indirect Trefftz approach for the analysis of both bounded and unbounded acoustic problems [22–24]. The wave functions that the indirect Trefftz approach used satisfy exactly the governing equations to describe the variables, and thus suit for the mid-frequency range. However, the high computational efficiency just exists for systems of moderate geometrical complexity and hence the use of the WBM is limited [25]. Obviously, attempts to improve the applicability of solutions based on WBM are required. A hybrid FE-WBM through combining the advantages of finite element and wave based techniques was proposed [25], which has drawn considerable attention. Lately, Zhang and Cheng presented a wavelet-Galerkin formulation to explore the general problem of internal sound field prediction and boundary shape optimization [26]. In summary, despite various methods developed for acoustic analysis of cavities, there is still a lack of comprehensive model for interior acoustic analysis, in particular in mid-frequency range, of the cavities considering such a complicated geometry as polyhedron which represents the generality of acoustic cavities in practical engineering, and also the impedance boundary.

The main aim of present paper is to furnish an efficient and powerful alternative to aforementioned analytical and numerical approaches, enabling the interior acoustic field and response of the irregular cavities like polyhedron with impedance boundary predictable in an effective and stable manner. In order to deal with the acoustic problem of the cavity with complex shape and accommodate the computation requirement of mid-frequency response, a decomposition technique is applied, namely, the cavity is divided into a proper number of sub-cavities. The impedance boundary condition is described by the work term in energy expression of all the sub-cavities. The Chebyshev orthogonal polynomials of the first kind (COPFK) are used as the basis function to globally expand the unknown sound pressure within the frame work of modified variational principle for each sub-cavity, and the required continuity conditions between any two adjacent ones are enforced by least-squares weighted residual method. It is worth noting that, when domain partitioning technique is not adopted, the formulation of present method could be reduced to that in Ref. [16]. However, differing from that in Ref. [16], the accuracy of sound pressure response results can be improved through increasing not only the polynomial order but also the number of the divided sub-cavities. The present approach has previously been used to solve the vibration problem of shell structures, showing that it is computationally accurate and efficient [27,28]. Therefore this study can be considered as an extension of the approach for structural vibration analysis to acoustics analysis. To demonstrate the accuracy and efficiency of the proposed approach, modal and frequency response analyses of cavities with different complicated geometries and wall impedances are investigated.

2. Theoretical formulations

2.1. Mathematical modeling

The considered acoustic cavity is enclosed by S ($S \geq 4$) quadrilateral walls. For example, a hexahedral-like cavity consists of 6 quadrilateral ones, of which each geometry can be characterized by the lengths of its four edges. Three angles ψ_1, ψ_2, and ψ_3 denote, respectively, the inclination of the ith wall about x-, y-, and z-axis of the rectangular coordinate system adopted for the mathematical modeling. Fig. 1 shows a cavity enclosed by 6 walls for the special case where $\psi_3 = 0$ and two of them are with inclined angles, ψ_1 and ψ_2, respectively, and the lengths of the cavity L_y, L_z are constant. This cavity is actually a rectangular-like cavity but with two arbitrary quadrilateral walls. If $\psi_1 = \psi_2 = 0$, $\psi_1 \neq 0$ and