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Dynamic stress concentration for multiple multilayered inclusions embedded in an elastic half-space subjected to SH-waves

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HIGHLIGHTS

- Dynamic stress concentration of multiple layered inclusions embedded within a half-space when subjected to an SH-wave is considered.
- A non-hypersingular boundary element method is employed to evaluate the stresses.
- The method is tested by exact analytical solutions.
- Numerical simulations reveal the effects of multiple scattering, layering, and impedance contrast of the layers on the stresses.

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ABSTRACT

The dynamic stress concentration factor (DSCF) is evaluated along the interfaces of multiple multilayered inclusions embedded in a half-space when subjected to a plane harmonic SH-wave. A weak form of Helmholtz equation is utilized to derive a non-hypersingular boundary integral equations to compute the stresses. Eliminating the need to rely on hypersingular integrals, greatly simplifies the procedure. The numerical results obtained by the proposed method, are validated against analytical solutions.

Various contributing factors that can influence the DSCF are investigated, including multiple scattering, layering, stiffness of the adjacent inclusions, and impedance contrast of the layers. The DSCF is found to be highly prone to these changes, particularly with the soft materials. Therefore, accurate analysis of stresses requires models that consider multiple scattering and layering. The presented result could be used for predicting the seismic failure of pipes and underground tunnels and for estimating the stress failure in strong ground motion seismology due to subsurface irregularities.

1. Introduction

The concentration of dynamic stresses in the scattering and diffraction of elastic waves plays a prominent role in many fields of science and engineering, including fracture mechanics, seismology, material science, non-destructive testing and the design of composites. The associated dynamic stress concentration factor can be obtained analytically or numerically. Analytical solutions for incident SH-waves are only available for simple geometries, such as a cavity in a full-space [1], a cavity in a half-space [2], and two circular cavities in a half-space [3]. Of the available numerical methods, the most prominent technique for the problem at hand is the boundary element method (BEM). A more detailed review of the BEM

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concerned with evaluation of the displacement field can be found in the authors' previous paper [4]. The focus of the present paper is boundary integral equation formulation for the evaluation of elastodynamic stresses.

The basis of the BEM is an appropriate fundamental solution, which, for the displacement fields, leads to strongly singular integral equations [4]. However, in computing the stresses, the kernels of the integral equations become hypersingular when the gradient of the displacement boundary integral equation (DBIE) is taken. To overcome this obstacle several techniques based on the integration by parts [5], Stokes' theorem [6], integral identities [7,8], and numerical methods using special quadratures [9] have been developed.

The basic idea behind the regularization of stress boundary integral equations (BIEs) involves writing the hypersingular kernel, which occurs in the integral equation, as a sum of singular and free terms. Consequently, it can be shown that the hypersingular integrals cancel out, leaving only weakly singular integrals that can be evaluated numerically [10–14]. For example, Guiggiani [10–12] demonstrated that the singular terms cancel out by considering the analytical limit and expanding the singular terms using local coordinates. Similarly, Gray [13,14] implemented a symbolic manipulator to take the analytical limit for the singular terms to show that they cancel out.

Krishnasamy et al. [6] invoked Stokes' theorem to express the singular surface integrals in terms of regular line integrals. This method enabled numerical evaluation of the required integrals.

Liu and Rizzo [7,8] presented a weakly singular form of the stress BIE using integral identities [15] of the fundamental solutions. They represented the displacement gradients as tangential derivatives, which required using the $C^{(1)}$ continuous elements.

Hildenbrand and Kuhn [9] employed Overhauser elements along with the Kutt's quadrature [16] to avoid the limiting process and to satisfy the $C^{(1)}$ continuity requirement.

One of the earliest analytical stress field results was obtained by Mow and Pao [1], who solved the scattering of a plane harmonic SH-wave by a circular cavity embedded in a full-space using the method of the wave function expansion. They computed the concentration of dynamic stress along the cavity for a wide range of frequencies.

Recently, numerical results based on BIEM were obtained for cavities and inclusions in a finite elastic solid when subjected to a plane harmonic SH-wave [17,18]. These results reveal the dependence of the stress concentration on the number, the location, and the shape of the scatterers. Although for the anti-plane strain model of the problem, the discretization of the half-space surface can be avoided by using the method of images, the aim of this paper is to develop a technique that can be extended to more general problems e.g. plane strain or 3D models for which the method of images is inapplicable. Consequently, the traction-free conditions on the surface of the half-space are imposed through discretization. This idea has been successfully used for other models by Niu and Dravinski [19] and Yu and Dravinski [20].

In the present investigation, the stress concentration factor is evaluated due to scattering of a plane harmonic SH-wave by multiple multilayered inclusions embedded in a half-space. The BIE is formulated using weak form of the equations of motion. This method was first suggested by Okada et al. [21–23] for elastostatics and later extended by Qian et al. [24] for problems in acoustics. The corresponding integral equation involves both weakly and strongly singular integrals. The strongly singular integrals can be expressed in terms of weakly singular integrals using the divergence theorem and different integral equation identities. This technique completely avoids evaluation of hypersingular integrals [24–26] and does not impose any continuity requirements for the discretization. Therefore, even linear elements can be used without relying on special numerical quadratures.

There are several practical problems for which this model can be used. For example, in strong ground motion seismology, one has to predict stress failure through the subsurface irregularities subjected to seismic waves. These irregularities may consist of a multiple scatterers of arbitrary shape with variable material properties. Therefore, the scatterers can be approximated by multilayered inclusions considered in this model. In addition, the present model can be applied to investigate seismic response of insulated pipes and underground tunnels. The pipes may contain different layers to prevent abrasion and corrosion. Similarly, tunnel linings can be made of different materials as well, involving concrete and steel. Of course, the application of the method to the realistic problems requires more general models, e.g. plane strain and 3D models. Therefore, the aim of the present paper is to develop a technique that can be readily extended to more general models for the problem under consideration.

2. Statement of the problem

The geometry of the problem is depicted by Fig. 1. The problem consists of multiple multilayered inclusions embedded in a half-space and subjected to a plane harmonic incident SH-wave. Here, S_j^i and D_j^i, $i = 0 : N, j = 0 : N$, are the layer interfaces and domains, respectively. The half-space surface and domain are denoted by $S^0 \equiv S_0^0$ and $D^0 \equiv D_0^0$, respectively. Furthermore, all of the displacements for domain D_j^i are represented by $u_j^{(i)}$. Consistent with the convention adopted earlier, the displacement field in the half-space is denoted by $u^0 \equiv u_0^0$.

The basic idea of the method is first outlined by considering scattering of plane harmonic SH-wave by single inclusion embedded in a full-space. These results are then generalized to the problem involving multiple multilayered inclusions embedded in a half-space.

The paper is organized as follows. First, the BIEs for the displacement gradients are introduced for the full-space problem. Next, these results are generalized to the corresponding half-space problem. Subsequently, the testing of the results is