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Theoretical study of the generation of screw dislocations for Rayleigh and Lamb waves in isotropic solids

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HIGHLIGHTS

- Screw dislocations appear on the surface of elastic solids by superposing SAWs.
- Phase singularities associated with Rayleigh or Lamb waves form hexagonal patterns.
- Displacement of singularities is possible for Lamb waves due to dispersion.

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ABSTRACT

Phase singularities are generic structures which occur in all wave fields, and they are characterised by an inability to assign a value to the phase. Screw dislocations are a particular kind of phase singularity where the phase possesses a helical structure, with the singularity at the centre of the helix. In this paper we show that it is possible to generate screw dislocations on the surface of elastic isotropic solids by means of the interference of three Rayleigh waves or three Lamb waves. The dispersive character of Lamb waves leads to more complicated behaviour, which may in turn result in greater potential for applications.

1. Introduction

A wave is defined by its amplitude, its phase and its polarisation. In special situations these quantities can be singular and lead to so called amplitude singularities, phase singularities and polarisation singularities [1]. Amplitude singularities arise on caustics where the amplitude of the wave field is singular according to the geometrical approximation. This case is of great interest and has been widely studied. The general framework is known as the theory of diffraction catastrophes [2]. Polarisation singularities concern only waves with polarisation. Phase singularities occur for waves when it is not possible to give a determined value to the phase. Two kinds of phase singularities exist [3]: edge dislocations and screw dislocations. Edge dislocations correspond to a straight singularity, whereas screw dislocations correspond to waves with a helical structure of phase. For this last configuration, the singularity is at the centre of the helix, where all the values of the phase are acceptable and thus no specific value can be assigned at this point. They have been mainly studied in optics, where they have motivated the birth of a new branch called "singular optics" [4]. They have also been studied for other kind of waves,

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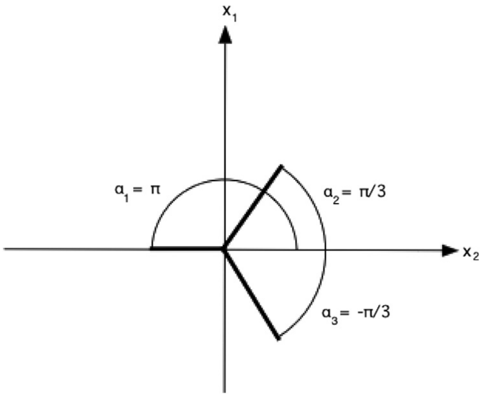

Fig. 1. Directions of propagation of three plane waves.

for instance tidal waves [5] or acoustic waves in fluids for linear [6] or nonlinear propagation [7–9]. Even if the extent of studies in acoustics is less important than in optics, it has been shown that phase singularities could be of great interest for different kind of applications. Indeed, screw dislocations or acoustical vortices possess a quantified pseudo angular momentum. This quantification offers the possibility of transmitting quantified information in the medium of propagation. Therefore, applications like imaging are possible [10]. The pseudo angular momentum also permits interaction with matter. It has been shown that it is possible to rotate absorbing disks [11–15] or even to build acoustical tweezers with acoustical vortices [16,17]. Theoretically, phase singularities occur for any kind of waves [18]. In this paper, we are interested in studying screw dislocations for elastic surface waves in isotropic solids. Surface Acoustic Waves (SAW) exist at different scales from geophysics to microfluidics [19]. As for acoustical vortices, we believe that surface screw dislocations could be used to probe the properties of the surface of materials or even to improve manipulation systems existing in microfluidics, for instance. Screw dislocations at the surface of isotropic and anisotropic solids exist and have already been observed [20]. In a recent paper, Riaud et al. [21] have proposed an original experimental setup to utilise swirling surface acoustic waves in order to produce acoustic vortices. In this paper we employ another approach to generate this kind of wave. Besides their existence (theoretically and experimentally proven), one important question has to do with the ability to generate this kind of feature of the wavefield. The aim of this paper is to propose a method to generate screw dislocations for Rayleigh and Lamb waves and also to analyse the positions and the behaviour of the screw dislocations depending on the frequency. The key idea is to produce a pattern of dislocations generated by the interference of different plane waves. This idea of superposing three wavefronts comes from optics, where the interference of three or more plane waves has been used to produce optical vortices by Masajada and Dubik [22] and O'Holleran et al. [23]. In the context of surface water waves, Karjanto and van Groesen [5] have shown that wavefront dislocation at singular points can be produced through the superposition of three monochromatic waves, and they have in fact demonstrated that three is the minimum number of waves necessary to give rise to such a dislocation. Because of the genericity of the screw dislocations [18], we expect this technique to be suitable for generation of singularities for SAW, provided the specific nature of these waves is taken into account. The general theoretical background is given in Section 2. Then, the method is applied to Rayleigh waves in Section 3 and to Lamb waves in Section 4. Similarities exist for these two kinds of waves but because of the dispersive character of Lamb waves, it is shown that behaviour of screw dislocations in this case is more complex and therefore seems more likely to pave the way to applications.

2. Phase singularities as a result of superposition of three wavefronts

Let us then consider three plane harmonic waves of equal amplitude travelling on the x_2–x_1 plane in different directions. The orientation of the x_2–x_1 axes is chosen so as to be consistent with the right-handed triad used in the following sections. We then represent each of these waves as

$$\psi_j(x_2, x_1) = e^{ik(x_2 \cos\alpha_j + x_1 \sin\alpha_j)}, \tag{1}$$

where $j = 1, 2, 3$, α_j is the polar angle, and k is a fixed wavenumber. Consider now an equilateral triangle placed in such a manner that its centre coincides with the origin of coordinates and one of its vertices lies along the negative x_2 axis. If we choose the wavefronts to propagate in the direction parallel to each of the bisectors of the triangle, we then have that $\alpha_1 = \pi$, $\alpha_2 = \pi/3$ and $\alpha_3 = -\pi/3$, as is shown in Fig. 1.

We will proceed to show that the resulting field contains an array of phase singularities, and then, furthermore, the localisation of these singularities can be explicitly found. Summing the three waves given in (1) we obtain

$$\begin{aligned}\psi(x_2, x_1) &= e^{-ikx_2} + e^{\frac{ik}{2}(x_2+\sqrt{3}x_1)} + e^{\frac{ik}{2}(x_2-\sqrt{3}x_1)} \\ &= e^{-ikx_2} + 2\,e^{ikx_2/2}\cos\left(\sqrt{3}\,kx_1/2\right),\end{aligned} \tag{2}$$