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Low-frequency wave propagation in an elastic plate floating on a two-layered fluid

D.A. Indejtsev [a], M.G. Zhuchkova [a], D.P. Kouzov [a], S.V. Sorokin [b,*]

[a] *Institute of Problems in Mechanical Engineering of Russian Academy of Sciences, St. Petersburg, Russia*

[b] *Department of Mechanical and Manufacturing Engineering, Aalborg University, Denmark*

H I G H L I G H T S

- Wave propagation in an elastic plate loaded with two-layered fluid of a finite depth is considered.
- Asymptotic analysis of the dispersion relation is done and several regimes of wave motion are identified.
- The thin layer approximation is derived.
- It is shown that the static pre-stress of a plate results in generation of waves with zero group velocity and triggers veering phenomenon.

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For some technical applications related to the ice–sea interaction, it is necessary to predict waveguide properties of elastic plates floating on a relatively thin layer of water, which has a non-uniform density distribution across its depth. The issue of particular concern is propagation of low-frequency waves in such a coupled waveguide. In the present paper, a stratified fluid is modelled as two homogeneous, inviscid and incompressible layers with slightly different densities. The lighter layer of fresh water lies on top of the heavier layer of salty water. The former one produces fluid loading at the pre-stressed plate, whereas the latter one is bounded by the sea bottom. The classical asymptotic methods are employed to identify significant regimes of wave motion in such a three-component waveguide. Dispersion diagrams obtained from approximate dispersion relations are compared with their exact counterparts. The phenomena of veering and generation of waves with zero group velocity induced by pre-stress are identified and quantified.

1. Introduction

Analysis of wave propagation in coupled waveguides is a canonical problem, which arises in many applications ranging from nano-mechanics to geophysics. One of such applications is the stationary fluid–structure interaction. Propagation of free and forced waves in fluid-loaded membranes and thin plates has been carefully studied in many research papers and the Refs. [1,2] are the most cited classical ones. The conventional formulation of a problem for a thin plate with fluid loading implies that the compressibility of a fluid is taken into account, and the fluid's volume is extended unboundedly in the direction perpendicular to the surface of a plate. The classical papers [1,2] and Refs. [3,4] provide a complete account of the phenomena occurring in this canonical waveguide.

* Corresponding author.
E-mail address: svs@m-tech.aau.dk (S.V. Sorokin).

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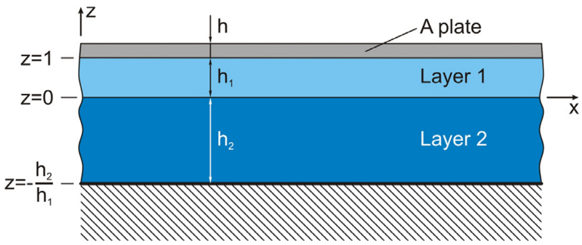

Fig. 1. The problem formulation.

A floating ice sheet constitutes a waveguide similar but not identical to the classical model of a fluid-loaded plate. Its waveguide properties have several important features, which are not captured in the above mentioned canonical problem formulation. First, the frequency range of interest is rather low, so that the compressibility effects are commonly neglected. On the other hand, it is essential that the density of an ice is less than that of water, and a plate floats at the surface of water. This requires modifications in the formulation of the interfacial conditions, which should account for the gravitational waves. Finally, it is a fairly common practise to consider in this context a fluid volume of the finite depth. Typically, a forcing problem, if formulated, addresses the interaction of an incoming from a fluid with free surface gravitational wave with a floating ice shield. The papers [5–8] and references wherein cover essential details of the fluid–structure interaction in such a formulation, specialized for the plane strain case.

The present paper takes these references as a point of departure, but advances in the following. In contrast to the above mentioned models, we consider the two-layer composition of fluid to account for stratification of water across its depth, which occurs due to ice melting in warm seasons. The simplification is that we assume a gradually inhomogeneous fluid being modelled as two homogeneous inviscid layers with slightly different densities. This model has been used in Ref. [9]. However, in [9] the free surface conditions for an upper layer have been considered. One more effect, which we take into consideration, is the static pre-stress of an elastic plate, typical for ice sheets. In contrast, the Ref. [10] deals with an elastic plate with no pre-stress floating on a single-layer fluid. Our goals are an asymptotic analysis of low-frequency travelling and evanescent waves and an assessment of validity ranges of simplified models for individual branches of dispersion diagram.

The paper is structured as follows. In Section 2, the governing equations for the model of a plate floating on a two-layered incompressible fluid are derived from the Hamilton's principle, and the dispersion equation is obtained. For consistency, the problem formulation for a conventional model of compressible fluid is presented in the Appendix and the comparison of dispersion diagrams for the frequency range of interest is done. Section 3 is concerned with an asymptotic analysis of roots of the dispersion equation with the special reference to low-frequency propagating waves. In Section 4, an asymptotic approximation for the dispersion relation in the case of a thin layer of upper fluid is derived. In Section 5, the effects of significant static pre-stress of a plate are analysed. The issues of particular interest are the generation of waves with zero group velocity and, due to the two-layer fluid composition, the associated veering phenomenon. The asymptotic formulae for evanescent waves are derived in Section 6. The limitations of the model employed in this paper and directions of further work are discussed in Section 7. The main results are summarized in Conclusions.

2. The variational problem formulation

We consider a two-layered fluid volume, shown in Fig. 1. It is bounded at one side by a rigid baffle and at the other side by a thin elastic plate. The thickness of a plate is h, the density of its material is ρ, its Young's module is E, and its Poisson ratio is ν. The plate is pre-stressed with intensity σ, it is of infinite extent in the x-direction (however, to derive the governing equations, we consider its part of the length L), and its lateral deflection is designated as $w\,(x, t)$. Fluid layers are characterized by densities ρ_n, sound velocities c_n and heights h_n, $n = 1, 2$. Their behaviour is described by the velocity potentials $\varphi_n\,(x, z, t)$. The gravity acceleration is g. Our concern is the low-frequency wave motion in this three-component waveguide, and, therefore, we adopt the model of an incompressible fluid. For consistency, the problem formulation with compressibility taken into account is presented in the Appendix. The plane problem formulation is used hereafter.

To derive the governing equations, we employ Hamilton's principle. The kinetic energy of the waveguide shown in Fig. 1 consists of:

- The kinetic energy of a plate:

$$T_{plate} = \frac{1}{2}\int_{(L)} \rho h \left(\frac{\partial w}{\partial t}\right)^2 dx. \tag{1}$$

- The kinetic energies of two layers of fluid:

$$T_{fluid} = \frac{1}{2}\int_{(L)}\int_{-h_2}^{z_f} \rho_2\,(\nabla\varphi_2)^2\,dzdx + \frac{1}{2}\int_{(L)}\int_{z_f}^{z_p} \rho_1\,(\nabla\varphi_1)^2\,dzdx. \tag{2}$$