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Line source representations for shear wave birefringence measurements in transversely isotropic materials using laser ultrasonics

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ABSTRACT

Line source representations for thermoelastic, laser excitation of transversely isotropic materials are developed to model shear wave birefringence effects that can occur in these materials. Continuum descriptions for elastic wave propagation are used to generate equations of motion that are solved using transform methods. Analytical solutions are presented for epicentral waveforms for propagation parallel and perpendicular to the axis of symmetry. These solutions are particularly simple and allow for rapid examination of the influence of various components of the stiffness tensor on wave propagation. In particular, for propagation perpendicular to the axis of symmetry, line source orientations parallel and perpendicular to this axis yield two cases that demonstrate shear wave birefringence. This effect is computed for titanium as well as for a nearly isotropic material – both Class I – to highlight the influence of diagonal as well as off-diagonal components of the stiffness tensor on the character of laser ultrasonic waveforms when the tensor is referenced to the principal coordinate system. The results presented in this work should be used to guide the interpretation of laser ultrasonic, line source measurements in transversely isotropic materials when birefringence effects are present.

1. Introduction

Elastic waves derived from line sources have received the attention of researchers dating back to Lamb whose classic work on step loading of an elastic half-space has inspired innumerable, succeeding investigations including the results presented here [1–3]. In this work, we develop models for ultrasonic waves produced using laser line sources in transversely anisotropic materials with the aim of developing specific shear wave polarizations that can be used to make birefringence measurements and characterize aspects of the material anisotropy. These models will use idealized line source representations for the laser source to simplify the overall treatment and to highlight the effects of elastic anisotropy on wave propagation. Taking this approach allows for a concise presentation of analytical solutions and isolates specific characteristics of these solutions. It also captures the essential physical processes permitting easy computation for comparison to experimental measurements. These comparisons are not presented in this work, but various characteristics of wave propagation that have not been explored previously will be noted — these potentially point to new directions for materials characterization using elastic waves.

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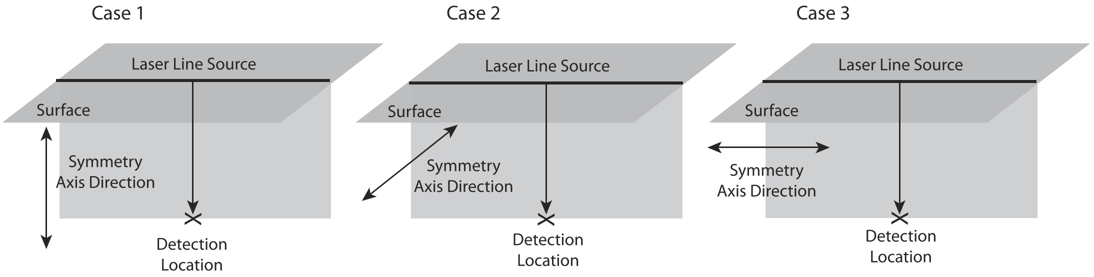

Fig. 1. Geometries for laser line source excitation of a transversely isotropic half-space. The three cases here illustrate the relationship of the surface- and source-orientation relative to the symmetry axis and also show the epicentral locations where displacements are evaluated.

An important characteristic of materials that directly impacts elastic wave propagation is texture — the partial alignment of elastically anisotropic grains/crystallites. For a population of randomly-oriented grains that interact with ultrasound over sufficient propagation distances, materials can behave isotropically. However, if the grains do not have complete orientational randomization in the sampled volume, then the material will be elastically anisotropic and wavespeeds will vary with direction and polarization [4–6]. Beyond grain orientation effects, partial alignment of defect structures such as microcracks can also induce elastic anisotropy [7–10]. Ultrasound has been used to assess this anisotropy and, with appropriate models, can be used to infer some characteristics of the underlying microstructure such as the orientation distribution coefficients. In particular, shear wave birefringence measurements can be used to isolate specific aspects of the anisotropy that can be useful for process monitoring and control [11–13] and various methods have been implemented to make these types of measurements [14,15], but none has taken advantage of laser line sources to produce polarized shear waves.

Laser line sources for elastic wave generation as well as the wavefields associated with these sources have been studied extensively. Early work simulated the behavior of thermoelastic sources using shear stress dipoles and focused on the directivity patterns associated with longitudinal and shear waves in isotropic materials [16]. Subsequent reports refined this general description by including details related to the underlying physics such as optical absorption, photothermal conversion and thermoelastic response [17–22]. Various approaches have been used to model characteristics of the resulting elastic wavefield and these have been verified experimentally [17]. One major finding is that the wave directivities and polarizations for these types of sources are complicated — even when the material is isotropic. Unlike other methods that have been used to perform ultrasonic shear birefringence measurements, the potential for using laser line sources for these types of measurements is largely unexplored. A good starting point for assessing this potential is to model shear wave birefringence in anisotropic materials excited using a laser source.

For isotropic materials, modeling of elastic waves generated by line sources has been the subject of many investigations. Indeed, Lamb's original problem is generally shown in textbooks on wave motion in elastic solids since it considers isotropic materials — the expressions for displacements in the wavefield are compact and readily interpreted [1,3,23]. The extension to anisotropic materials and point sources requires effort but yields rich results that go well beyond those to be exploited here [24–31]. Fortunately, line source excitation of transversely anisotropic half-spaces has also been considered in some detail and the essential mathematical techniques required to derive solutions are generally no more involved than those used for isotropic materials [32–34]. However, compact analytical solutions exist only for specific cases, but these can be obtained using careful algebraic manipulations. While direct numerical solution of the governing equations and boundary conditions can be performed [35,36], simple insights into the overall behavior of the system can be lost — analytical results permit general conclusions to be made that might be difficult to identify otherwise. The subsequent sections will develop models for the epicentral displacements resulting from laser line excitation of transversely isotropic half-spaces of different orientations. The development will largely follow approaches presented previously by Payton [37], Hurley [38] and others [39] with minor changes to notation to improve clarity. Three specific cases will be considered: Case I — Line source in the plane of symmetry, displacements along the symmetry axis; Case II — Line source perpendicular to the symmetry axis, displacements perpendicular to the symmetry axis; Case III — Line source parallel to the symmetry axis, displacements perpendicular to the symmetry axis. The geometry for each is shown in Fig. 1. While only Cases II and III are needed to illustrate shear wave birefringence, Case I shows the methods used for solution and complements the other results. In all cases, final solutions for epicentral displacements will be given for materials having convex normal surfaces (Class I materials) such as occurs for various single crystals with hexagonal symmetry [37] as well as for certain weakly-textured, polycrystalline materials displaying transverse isotropy.

2. Theory and models

In the following developments, epicentral displacements from suitably-oriented, unit strength, shear stress dipoles will be derived. These stresses relate directly to the corresponding surface tractions that are appropriate for each case. The relationship of shear stress dipoles to laser line sources has been established previously and these are relatively simple