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The analytical solutions for the wave propagation in a stretched string with a moving mass

Q. Gao *, J. Zhang, H.W. Zhang, W.X. Zhong

State Key Laboratory of Structural Analysis of Industrial Equipment, Department of Engineering Mechanics, Faculty of Vehicle Engineering and Mechanics, Dalian University of Technology, Dalian 116023, PR China

HIGHLIGHTS

- The analytical solutions are derived for a mass moving along a stretched string.
- The solutions cover the string of infinite, semi-finite and finite length.
- A mass moving at subsonic, sonic or supersonic velocities are all considered.

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ABSTRACT

This paper derives the analytical solutions for a stretched string subjected to a concentrated mass moving at a constant velocity. From the derived analytical solutions of the contact force between the string and the mass, the displacement responses of the string can be easily obtained. The solutions cover an infinite, semi-infinite or finite string subjected to a moving mass at subsonic, sonic or supersonic velocities. For the semi-infinite or finite strings, the solutions for different types of boundary conditions are presented in both a unified form and in the form of a series of exponential and polynomial functions. The formula derived is shown to be correct by comparison with the semi-analytical method.

1. Introduction

The interaction between a moving force or mass and a structure is comprehensively investigated in references [1,2]; a string subjected to a moving mass is one of these investigated problems. The string model can be applied to simulate engineering problems, such as the contact wire in a coupled pantograph–catenary system [3] and some aerospace structures [4]. The dynamic responses of the string subjected to the moving force or mass can be obtained using analytical or numerical methods.

By using the Laplace transformation, Kanninen and Florence [5] derived an analytical solution for a stretched infinite string subjected to a moving force at a constant speed, and a set of three solutions was obtained for subsonic, sonic and supersonic velocities. Langlet et al. [6] also obtained the exact solution for the same problem. A stretched infinite string excited by a moving force with a varying speed is investigated in references [7,8]. Based on the Laplace and Fourier transformations, a stretched semi-infinite string subjected to a moving force with a constant acceleration was considered by Sagartz and Forrestal [9]. Based on the Laplace transformation, the dynamic response of an inhomogeneous, stretched infinite string on

* Corresponding author. Tel.: +86 411 84707608.
E-mail addresses: qgao@dlut.edu.cn (Q. Gao), dl_zhangjian@163.com (J. Zhang), zhanghw@dlut.edu.cn (H.W. Zhang), zwoffice@dlut.edu.cn (W.X. Zhong).

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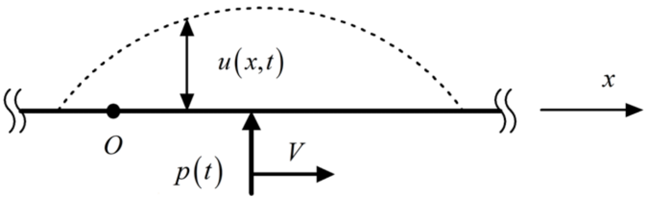

Fig. 1. An infinite string subjected to a moving force.

an elastic foundation subjected to a moving force at a constant speed was investigated by Wolfert et al. [10]. Dieterman and Kononov [11] used Fourier transformations to investigate the dynamic response of a stretched infinite string on an elastically supported membrane subjected to a moving force. Oniszczuk [12] used the mode superposition principle to calculate the dynamic response of a stretched finite double string that was connected to a Winkler elastic layer subjected to a moving harmonic force at a constant speed. Rusin and Sniady [13] then presented a closed-form solution to the same problem. In references [5,7–15], the moving force is considered as the only external excitation, and the interaction between the moving mass and string is not presented.

Smith [16] presented an analytical solution for a stretched finite string subjected to a mass moving at a constant speed, but the mass of the string was neglected in this model. Rodeman et al. [4] derived a numerical solution for a stretched semi-infinite string subjected to a constantly accelerating moving mass, and an asymptotical solution was also obtained when the effect of the inertia of the moving mass was small. Wang and Rega [17] used the Hamilton's principle to derive the 3D nonlinear equations of motion of a suspended cable subjected to a moving mass, and then applied the Newmark method to calculate the dynamic response of the suspended cable. Yang et al. [18] obtained an integral equation for the response of a stretched finite string subjected to a moving mass at a constant speed and then used a numerical integration method to solve this equation. By using a combination of the Fourier integral transformation and time integration methods, Dyniewicz and Bajer [19] proposed a semi-analytical method for the solution of a stretched finite string subjected to a moving mass at a constant speed. The researchers then presented a space–time method for obtaining a numerical solution to the same problem [20]. Kruse et al. [21] introduced a moving coordinate system to obtain the eigenfrequencies of a stretched infinite string on a viscoelastic foundation subjected to a two-mass oscillator at a constant speed. Lee et al. [22] adopted the Lagrange multiplier method to describe the dynamic responses of a coupled moving mass-stretched beam with a separation between each mass. Gavrilov [23] investigated a moving mass on a string resting on a Winkler foundation by also considering the wave drag. Bajer and Dyniewicz [24] presents many numerical methods and semi-analytical solutions to solve the problems of the vibrations of structures subjected to moving loads or masses.

To the authors' knowledge, there is no analytical solution for a stretched string whose mass is not negligible and that is subjected to a moving mass. The discovery of any explicit solutions whatsoever is of great interest. The solutions can be used as models for physical experiments and as benchmarks for testing numerical methods, for example. In this paper, the analytical solutions for a stretched string with a mass moving at constant velocity are considered. The solutions cover the scenarios of an infinite, semi-infinite or finite string subjected to a mass moving at subsonic, sonic or supersonic velocities. For a semi-infinite or finite string, the solutions for different types of boundary conditions are also considered.

2. The analytical solutions for a stretched string with a moving load

In this section, the analytical solutions for a stretched string (infinite, semi-infinite or finite) subjected to a force moving at constant speed (subsonic, sonic or supersonic) are presented. Those solutions are classical solutions and can also be found in other references; however, these solutions are necessary for the derivation of the main solutions presented in this paper and are therefore provided in detail.

As shown in Fig. 1, let us consider a string of infinite length, line mass density ρ and tensile force T subjected to a concentrated force $p(t)$ that moves in the positive x direction, starting from the original point O, at a constant speed V. The motion equation of the string can be written as

$$\rho\frac{\partial^2 u}{\partial t^2} = T\frac{\partial^2 u}{\partial x^2} + p(t)\,\delta(x - Vt), \quad -\infty < x < +\infty,\ t > 0 \tag{1}$$

where δ denotes the Dirac delta function. The initial conditions are

$$u(x, 0) = \frac{\partial u}{\partial t}(x, 0) = 0. \tag{2}$$

Eq. (1) can also be written as

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2} + \rho^{-1}p(t)\,\delta(x - Vt) \tag{3}$$