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Modeling non-collinear mixing by distributions of clapping microcracks

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HIGHLIGHTS

- Non-collinear mixing by distributions of clapping microcracks is modeled.
- Constitutive relationships for distributions of clapping cracks are presented.
- Linear dependence of scattered waves on the incident field is found.
- Selection rules for nonlinear scattering by quadratic nonlinearity are violated.
- Examples of forward and backscattering for discrimination purposes are presented.

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ABSTRACT

Nonlinear scattering by distributions of clapping cracks in a non-collinear wave mixing setting is modeled. Features of the nonlinear response discriminating distributions of clapping cracks from quadratic nonlinear damage are investigated for distributions of cracks that are parallel to each other or randomly oriented. The effective properties of these distributions are recovered extending an existing model that applies to open cracks. The equation of motion is solved using a perturbation approach, and its solutions are evaluated numerically. Their dependence on the amplitude of the incident field is found to be linear, in contrast with the quadratic dependence characterizing quadratic nonlinearity. The spectrum of the scattered field is shown to contain an infinite number of higher harmonics already at the first order of perturbation. Grating-like structures due to the opening and closing of cracks are responsible for adding diffraction peaks to the directivity functions of waves scattered by open cracks. The locations of the most prominent peaks of these functions do not satisfy the selection rules controlling nonlinear scattering by quadratic nonlinearity. Examples of these are given, together with others showing the possibility of using at least one of several discrimination modalities offered by non-collinear wave mixing.

1. Introduction

The onset of damage in engineering components is accompanied by the appearance of nonlinearities in their mechanical properties. In Earth and similar man-made materials, on the other hand, nonlinearity is a permanent feature caused by defects like dislocations, grain boundaries, or microcracks, which are inherent components of the microstructure of this class of materials.

When a material is excited by a stress wave, mechanical nonlinearity may manifest itself in several ways. Generation of the second harmonic component during propagation of a monochromatic wave is the one that has been most investigated

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because, perhaps, it is the most common nonlinear response of materials of engineering interest (see review article by Matlack et al. [1] and references therein). The nonlinear response of Earth materials, on the other hand, is more complex. In addition to quadratic nonlinearity, dynamic effects of mechanical hysteresis with end-point memory may be observed. Among these, one finds the preferred generation of odd higher harmonics and of side-band spectral components, the negative shift of resonance frequencies corresponding to material softening, and nonlinear energy dissipation. Depending on the experimental technique employed in a given investigation and on the physical origin of the nonlinearities, these phenomena may or may not appear simultaneously [2].

Of interest to the present communication is an acoustic technique known as non-collinear wave mixing. Non-collinear wave mixing uses two non-parallel, coplanar beams to interrogate a region of interest (RoI) in a material sample [3,4]. The RoI is defined by the volume over which the acoustic paths of the two beams intersect each other. In it, waves interact with one another only if the properties of the material are nonlinear. This interaction is responsible for the generation of waves with polarization, frequency, and wave vectors that may differ from those of the incident fields and of other linear components. These specific properties of the nonlinear fields are determined by the physical nature of the material nonlinearity, and, thus, can be used to enhance the detectability of these waves and for characterization purposes. For instance, the frequencies of the incident waves may be chosen so that those of the nonlinear components fall in a range in which there is no other signal (frequency discrimination). Similarly, the wave vectors of the incident beams can be selected so that the nonlinear scattered components propagate along directions which eventually lead them to be spatially separated from the incident and linearly scattered waves (spatial discrimination). Finally, nonlinear scattered waves can be generated having polarization which differs from that of the interrogating waves (mode discrimination).

Collinear wave mixing is used when the geometry of the component under inspection renders the use of a two-dimensional configuration of the inspection set-up impossible (see [5] and references within). In this case, two sensors opposing each other and working in transmission, or assembled in a single unit and working in reflection mode are utilized. In both cases, two beams are generated, which have spatially coincident acoustic paths. Therefore, collinear wave mixing does not offer the same degree of flexibility and power of discrimination as the non-collinear technique.

To the best of this author's knowledge, mixing of non-collinear waves has been investigated theoretically only in materials with quadratic nonlinearities [3]. Characteristic features of the nonlinear waves scattered by quadratic nonlinear media are the generation of waves with frequencies equal to the sum and difference of the frequencies of the incident waves. Necessary conditions for the generation of these nonlinear waves is the satisfaction of one of the following four selection rules

$$\vec{k}_1 \pm \vec{k}_2 = \left[(\omega_1 \pm \omega_2)/C_{L,T}\right]\vec{r}_{L,T}. \tag{1}$$

In Eq. (1), $\vec{k}_1$ and $\vec{k}_2$ are the wave vectors of the incident beams, ω_1 and ω_2 their frequencies, $C_{L,T}$ the phase velocities of longitudinal and shear waves, and $\vec{r}_{L,T}$ the unit vectors identifying the directions, if they exist, along which longitudinal or shear waves satisfy one of these four equalities. Satisfaction of these conditions guarantees that the phase of the mixed terms, which arise from the product of derivatives of the particle displacement components, is zero. Direct consequence of this fact is that certain integrand functions no longer depend on the integration variable, making the value of the integrals proportional to the volume of integration.

Quadratic nonlinearity may arise because of more than one reason. Jones and Kobett [3] examined the effect of nonlinearity introduced in the equation of motion by retaining third order terms in the particle displacement in the material's elastic potential energy. Plastic and fatigue damage [6,7] were also shown to generate a nonlinear response that follows the predictions in [3]. Bowing of pinned dislocations is a possible mechanism at the atomic scale which contributes under such circumstances (see for example [8,9] and references therein). Finally, Jiao et al. [10] used the collinear wave mixing technique to detect microcracks under the assumption that their response to a driving acoustic field is a quadratic function of the latter. Distributions of microcracks can be found in Earth and similar man-made materials, and in steel components of nuclear reactor vessels that have been exposed to intense neutron radiation. The coalescence of atomic-scale defects, like vacancies first and dislocation loops at a later stage, can eventually produce distributions of microcracks [11].

It follows from the brief review of some relevant literature that non-collinear wave mixing is not capable of discriminating forms of damage from each other as long as their responses are quadratic in the strain field. The identity of responses by drastically different defects like dislocations and weak boundaries on one side, and microcracks on the other would appear to be a clear drawback of this technique. In fact, profoundly dissimilar assessments of the properties of a material's microstructure may be arrived at depending on whether an inspection has revealed its structure to contain, for example, distributions of microcracks or dislocations. Therefore, a different response should be elicited from either one of them if their physical nature is to be identified by means of this technique. The present work addresses this problem by analyzing the nonlinear response of distributions of microcracks that is not quadratic.

The quadratic nonlinear response of a crack is mostly due to the linear dependence of its finite stiffness on the amplitude of the incident wave. The finite stiffness of a crack may be the consequence of its faces being in partial contact due to their non-conformity, a situation common to fatigue and stress corrosion cracks. Another cause for the stiffness of a crack to vary with the amplitude of the incident field may be the modulation of the area over which the crack is entirely open. Cracks, which completely open or close under the action of an incident wave, transitioning from one state to the other as soon as the normal component of traction vector changes direction, are said to be "clapping". Complete closure of a crack is possible