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Green's function retrieval in a field of random water waves

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HIGHLIGHTS

- Water wave interferometry is described theoretically and investigated experimentally.
- Simulations and wave tank measurements are in agreement with theoretical predictions.
- Ocean-based measurements are not in agreement with theoretical predictions.
- Probable reasons for failure of the ocean-measurement-based analysis are discussed.

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ABSTRACT

It has recently been demonstrated using a variety of wave types that cross-correlating time series of apparently random waves measured at two locations yields an estimate of the Green's function that describes the wave field generated at one of those locations and measured at the other. This procedure can be described as random wave interferometry. In this paper random surface gravity wave interferometry is described theoretically, demonstrated using numerical simulations, and investigated experimentally using both wavetank measurements and ocean wave measurements. Simulations and wavetank measurements are in good agreement with theoretical predictions, but the ocean-measurement-based cross correlations do not yield the predicted structure. Possible explanations are discussed.

1. Introduction

In recent years it has become appreciated that deterministic wave propagation information can be extracted from an apparently random wave field via a process that can be described as random wave interferometry; the cross-correlation of measurements at two locations of a random wave field yields an approximation to the Green's function that describes propagation between those locations [1–13]. Random wave interferometry has been widely investigated in the context of elastic waves in solids [14–16], including seismic [17–22] and helioseismic [23,24] applications, and sound waves in fluids, including applications to ocean acoustics [25–35] and atmospheric acoustics [36–38]. The underlying theory is widely applicable, as it can be applied to any type of linear wave propagation.

In this paper we study, both theoretically and experimentally, the application of these ideas to water waves, i.e., surface gravity waves. This topic can be referred to as random water wave interferometry. Previous work on random water wave interferometry is described in [39,40]. [39] focuses on theory relating to tsunamis (very low frequency ocean surface gravity waves for which $kh \ll 1$), while [40] presents theory and data analysis relating to infragravity waves (low frequency ocean surface gravity waves for which $kh = O(1)$ in the deep ocean). The latter work builds on the earlier work [41] which demonstrated that seafloor measurements of infragravity-wave-induced pressure fluctuations separated by a few 10's of km are coherent, and the work reported in [42], which demonstrated the utility of cross-correlating measurements of that type.

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The basic result on which we focus in this paper is

$$\gamma C_{AB}(t) = D(t) * [G_\eta(\mathbf{x}_B|\mathbf{x}_A, t) + G_\eta(\mathbf{x}_A|\mathbf{x}_B, -t)]. \quad (1)$$

Here γ is a constant, $C_{AB}(t)$ is the correlation function of records of surface elevation $\eta(t)$ at locations $\mathbf{x}_A$ and $\mathbf{x}_B$, $D(t)$ is a band-limited approximation to a delta function, and $G_\eta(\mathbf{x}|\mathbf{x}_0, t)$ is the transient surface elevation at point $\mathbf{x}$ due to an impulsive point source at $\mathbf{x}_0$. Two derivations of Eq. (1), which holds in both open and closed systems, are given below. It is important to appreciate that two results underlie Eq. (1) — an exact identity involving Green's functions, and a strong approximate statistical assumption. The Green's function identity holds provided the underlying fluid dynamical assumptions (adequacy of linear theory, including the assumption that the flow remains irrotational) are satisfied. The statistical assumptions underlying Eq. (1) are strong and might reasonably be questioned, but we note that experience in other fields has shown that relationships similar to Eq. (1) are good approximations even when the underlying statistical assumptions are only approximately satisfied.

The remainder of this paper is organized as follows. The theory underlying Eq. (1) is presented in Section 2. Derivations of Eq. (1) for both open and closed systems are included. In Section 3 simulations, in both open and closed systems, are presented and shown to be consistent with Eq. (1). In Section 4 the analysis of data collected in a wavetank for the purpose of testing Eq. (1) is described. Agreement between theory and measurements is good. In Section 5 the analysis of a set of ocean wave measurements, again to test Eq. (1), is described. Those results are not in good agreement with theory. In Section 6 our results are discussed and summarized.

2. Theory

In this section we present two derivations of Eq. (1), which holds in both open and closed systems. Each derivation makes use of an identity involving Green's functions, Eq. (12) or (22), and a strong statistical assumption, Eq. (14) or (24).

2.1. Preliminaries

We begin with comments on two topics. First, it is important to distinguish between two different Green's functions: G_ϕ is the velocity potential Green's function evaluated at the undisturbed free surface; and G_η is the surface displacement Green's function. Both of these quantities have time- and frequency-domain representations. Second, convolutions and cross-correlations play important roles in the derivations below. Consider real-valued functions $f_1(t)$ and $f_2(t)$. Convolution of $f_1(t)$ and $f_2(t)$ is defined as $f_1(t) * f_2(t) = \int_{-\infty}^{\infty} d\tau f_1(\tau) f_2(t-\tau)$. The cross-correlation of $f_1(t)$ and $f_2(t)$ is defined as $f_1(t) * f_2(-t) = \int_{-\infty}^{\infty} d\tau f_1(\tau) f_2(t+\tau)$. Let $\bar{f}(\omega) = F[f(t)] = \int_{-\infty}^{\infty} f(t) e^{i\omega t} dt$ denote the Fourier transform of $f(t)$, and let $f(t) = F^{-1}[\bar{f}(\omega)]$ denote the inverse transform. Because these variables are real-valued, it is assumed that $\bar{f}(-\omega) = \bar{f}^*(\omega)$, where the superscript * denotes complex conjugation. The Fourier transform of the convolution $f_1(t) * f_2(t)$ is $\bar{f}_1(\omega)\bar{f}_2(\omega)$, and the Fourier transform of the cross-correlation $f_1(t) * f_2(-t)$ is $\bar{f}_1(\omega)\bar{f}_2^{\,*}(\omega)$. For the purpose of computing the cross-correlation of two measured time series it is necessary to replace the unbounded integration domain with a finite domain. This leads a modified definition of the cross-correlation, $f_1(t) \star f_2(t) = \int_0^T d\tau f_1(\tau) f_2(\tau + t)$. The $*$ and $\star$ notation introduced here is used below. It is often convenient to normalize $f_1(t) \star f_2(t)$ by dividing by T; when this is done, division by T will be shown explicitly.

The starting point of our analysis is the linearized water wave equations of motion (see, e.g., [43]). Let $\mathbf{x} = (x, y)$ denote lateral position vector, and assume that the vertical coordinate z increases upwards, with $z = 0$ at the undisturbed free surface. Let ∇^2 denote the three dimensional Laplacian operator, and let $\nabla_\perp$ and $\nabla_\perp^2$ denote the two-dimensional – in (x, y) – gradient and Laplacian operators, respectively. We assume that the water depth h is constant. Let $\Phi(\mathbf{x}, z, t)$ denote the velocity potential and $\bar{\Phi}(\mathbf{x}, z, \omega)$ its Fourier transform. The problem is to solve $\nabla^2 \Phi(\mathbf{x}, z, t) = f(\mathbf{x}, z, t)$ subject to $\partial^2\Phi/\partial t^2 + g\partial\Phi/\partial z = 0$ at $z = 0$, and $\partial\Phi/\partial z = 0$ at $z = -h$. The free surface displacement $\eta(\mathbf{x}, t)$ is equal to $(-1/g)\partial\Phi/\partial t$ evaluated at $z = 0$. $f(\mathbf{x}, z, t)$ is a source function; ρf is the rate at which mass per unit volume is injected. The problem as defined so far is poorly posed. The difficulty is that most functions $f(\mathbf{x}, z, t)$ do not couple naturally to small amplitude surface gravity waves, and therefore do not allow the specified free surface and bottom boundary conditions to be satisfied. Instead, most functions $f(\mathbf{x}, z, t)$ lead to a localized transient non-wave-like response. To avoid this difficulty, we shall assume that the depth dependence of the source function $f(\mathbf{x}, z, t)$ is chosen to couple naturally to small amplitude surface gravity waves. (It is worth noting that the same assumption is made below in a less obvious way when we find the surface displacement Green's function as the solution to an initial value problem; in both cases the depth dependence of the velocity field is constrained to be dynamically consistent with the free surface displacement.)

In the frequency domain, the problem is to solve $\nabla^2\bar{\Phi}(\mathbf{x}, z, \omega) = \bar{f}(\mathbf{x}, z, \omega)$ subject to the boundary conditions $\omega^2\bar{\Phi} = g\partial\bar{\Phi}/\partial z$ at $z = 0$, and $\partial\bar{\Phi}/\partial z = 0$ at $z = -h$. We shall assume that $\bar{f}(\mathbf{x}, z, \omega) = \bar{p}(\mathbf{x}, \omega)\cosh k(z+h)/\cosh kh$ and that $\bar{\Phi}(\mathbf{x}, z, \omega) = \bar{\phi}(\mathbf{x}, \omega)\cosh k(z+h)/\cosh kh$. With these assumptions the boundary conditions are satisfied provided the dispersion relation $\omega^2 = gk\tanh kh$ is satisfied, and the problem is reduced to solving $(\nabla_\perp^2 + k^2)\bar{\phi}(\mathbf{x}, \omega) = \bar{p}(x, \omega)$. The solution can be written $\bar{\phi}(\mathbf{x}, \omega) = \iint d\mathbf{x}' \bar{G}_\phi(\mathbf{x}|\mathbf{x}', \omega)\bar{p}(\mathbf{x}', \omega)$ where $\bar{G}_\phi(\mathbf{x}|\mathbf{x}', \omega)$ is the Green's function for the Helmholtz equation in two space dimensions,

$$(\nabla_\perp^2 + k^2)\bar{G}_\phi(\mathbf{x}|\mathbf{x}', \omega) = \delta(\mathbf{x} - \mathbf{x}'). \quad (2)$$