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Single slit diffraction: From optics to elasticity

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HIGHLIGHTS

- Results of simulations of plane wave propagation through a single slit are presented.
- Classical optical, acoustic, and elastic cases are compared.
- Elastic case differs from optical and acoustic ones.
- The variation of material parameters changes the stress distribution drastically.
- The observed difference between acoustic and elastic cases is unavoidable.

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ABSTRACT

The comparison of numerical simulations of the classical problem of the single slit diffraction in optical, acoustic, and elastic cases is presented in the paper in the plane strain setting. It is shown that wave fields downstream the slit are similar in optical and acoustic cases, as expected. Corresponding wave fields in the single slit diffraction using elastic materials become essentially different from optical and acoustic cases. This is an effect of elastic waves propagating inside the plate forming the slit.

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1. Introduction

Diffraction is a well studied phenomenon, especially in classical optics since the celebrated Tomas Young double slit diffraction experiment [1] demonstrated the wave-like behavior of light. In elasticity, much attention was paid to scattering problems due to their practical application [2,3]. Since diffraction and scattering are complementary phenomena, the same mathematical technique is used in their description, especially in acoustics, because of identity of the wave propagation equation in acoustics and classical optics [4]. The conversion of longitudinal and shear waves at boundaries in scattering and diffraction problems in elasticity makes the corresponding solution much more complicated.

The emerging field of metamaterials (composites with unusual macroscopic properties due to local resonances) provides an unprecedented way for controlling wave propagation in a desired way by tailoring the microstructure [5]. Simultaneously, it demands a more precise prediction of the wave field. The problem of wave propagation through heterogeneous materials has been considered since the mid-nineteenth century [6], and since then the progress in considering ever more complicated scattering structures has been continuous [7–9]. Various approaches to predicting the wave propagation through different scattering structures have recently been reviewed by Martin [10] and Harris [11].

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Analytical difficulties often lead to the restriction of the analysis by limiting cases of very short wavelengths (ultrasound) and of very long wavelengths (quasi-statics). It is remarkable that the elastic analogue of the Talbot effect known in classical optics for a long time [12] has been demonstrated only recently [13]. The reason is in the comparability of wavelengths with the slit size of the grating in the elastic case.

The prediction of the wave field is the key element in the wave control. The growth of computer power and the progress in numerical methods provide a direct numerical solution of diffraction problems. The major advantage of numerical simulation is its generality and the capability of predicting wave fields for any composite with arbitrarily distributed scatterers.

Before the application of numerical simulations to complicated situations, it is instructive to start with one of the basic problems—the single slit diffraction. This problem is studied in detail in classical optics both theoretically and experimentally [14]. Classical optics is characterized by the extremely short wavelength in comparison to the slit size, while in acoustics the wavelength and the slit size may have the same order of magnitude. The absence of shear waves is common for both optics and acoustics. It should be noted, however, that traditionally the plate forming a slit is opaque (not transparent) in optics and perfectly rigid in acoustics. The completely elastic formulation of the single slit diffraction suggests the elastic behavior both for the matrix and for the plate forming the slit.

In the paper, we try to emphasize the similarity and the difference between the single slit diffraction in optics, in acoustics, and in elasticity demonstrating numerically calculated wave fields. The major difference of the elastic case from the classical single slit diffraction in optics and acoustics is due to the propagation of wave through the matrix as well as through the plate forming the slit, which is not present in classical optics and in acoustics case. Additionally, both longitudinal and shear waves are accounted for in the elastic case, while in acoustics only longitudinal waves are taken into account. The variation of the thickness of the plate forming the slit is also analyzed. This will permit us later to understand better the influence of geometrical shapes of gratings and possible nonlinearities of materials.

The paper is organized as follows. In Section 2 we introduce the governing equations for the plain strain elasticity and present their non-dimensional form in Section 3. In Section 4 numerical results for various cases are presented based on applying the modified wave-propagation algorithm [15,16]. The corresponding governing equations are specified with suitable scaling and assumptions. Section 5 includes conclusions and some ideas for further studies.

2. Plane strain elasticity

Numerical simulation of elastic wave propagation is based on the solution of equations of elasticity. Although the governing equations are well-known, we represent here the basic forms in order to explain later the possible simplifications. Neglecting both geometrical and physical nonlinearities, we can write the bulk equations of homogeneous linear isotropic elasticity in the absence of body force as follows [17]:

$$\rho_0 \frac{\partial v_i}{\partial t} = \frac{\partial \sigma_{ij}}{\partial x_j}, \tag{1}$$

$$\frac{\partial \sigma_{ij}}{\partial t} = \lambda \frac{\partial v_k}{\partial x_k}\delta_{ij} + \mu \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right), \tag{2}$$

where t is time, x_j are spatial coordinates, v_i are components of the velocity vector, σ_{ij} is the Cauchy stress tensor, ρ_0 is the density, λ and μ are the Lamé coefficients.

Consider a sample that is relatively thick along x_3, and where all applied forces are uniform in the x_3 direction. Since all derivatives with respect to x_3 vanish, all fields can be viewed as functions of x_1 and x_2 only. This situation is called plane strain. The corresponding displacement component (e.g., the component u_3 in the direction of x_3) vanishes and the others (u_1, u_2) are independent of that coordinate x_3; that is,

$$u_3 = 0, \qquad u_i = u_i(x_1, x_2), \quad i = 1, 2. \tag{3}$$

It follows that the strain tensor components, ε_{ij} are

$$\varepsilon_{i3} = 0, \qquad \varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad i, j = 1, 2. \tag{4}$$

The stress components follow then

$$\sigma_{3i} = 0, \qquad \sigma_{33} = \frac{E}{1-2\nu}\left(\frac{\nu}{1+\nu}\varepsilon_{ii} \right), \quad i = 1, 2. \tag{5}$$

$$\sigma_{ij} = \frac{E}{1+\nu}\left(\varepsilon_{ij} + \frac{\nu}{1-2\nu}\varepsilon_{kk}\delta_{ij} \right), \quad i, j, k = 1, 2, \tag{6}$$

where E is Young's modulus, ν is Poisson's ratio, δ_{ij} is the unit tensor.

Inversion of Eq. (6) yields an expression for the strains in terms of stresses:

$$\varepsilon_{ij} = \frac{1+\nu}{E}\left(\sigma_{ij} - \nu\sigma_{kk}\delta_{ij} \right), \quad i, j, k = 1, 2. \tag{7}$$

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