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Simulation of elastic wave propagation using cellular automata and peridynamics, and comparison with experiments

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HIGHLIGHTS

- Peridynamic and Cellular Automata 2D-elastodynamic responses are evaluated.
- Each method is subjected to a normal load on a half-plane (Lamb's Problem).
- Cellular Automata and Peridynamics reproduce the shape and location of the pressure, shear and Rayleigh wave very well.
- Neither method is able to follow experimental results.

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ABSTRACT

Peridynamics is a non-local continuum mechanics formulation that can handle spatial discontinuities as the governing equations are integro-differential equations which do not involve gradients such as strains and deformation rates. This paper employs bond-based peridynamics. Cellular Automata is a local computational method which, in its rectangular variant on interior domains, is mathematically equivalent to the central difference finite difference method. However, cellular automata does not require the derivation of the governing partial differential equations and provides for common boundary conditions based on physical reasoning. Both methodologies are used to solve a half-space subjected to a normal load, known as Lamb's Problem. The results are compared with theoretical solution from classical elasticity and experimental results. This paper is used to validate our implementation of these methods.

1. Introduction to peridynamics

Peridynamics (PD) is a non-local continuum mechanics theory that avoids spatial derivatives in favor of integro-differential equations that was introduced by Silling in 2000 [1]. The nonlocality of PD means that deformation at a point is influenced by all points in a region surrounding this point.

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The governing equation of bond-based PD is given as:

$$\rho(\boldsymbol{x})\ddot{\boldsymbol{u}}(\boldsymbol{x},t)=\int_{H_x}\boldsymbol{f}(\boldsymbol{u}'-\boldsymbol{u},\boldsymbol{x}'-\boldsymbol{x})\mathrm{d}V'+\boldsymbol{b}(\boldsymbol{x},t) \tag{1}$$

where ρ is the mass density, $\boldsymbol{x}$ is the position, $\boldsymbol{u}$ is the deformation, t is the time, $\boldsymbol{f}$ is the force density between points $\boldsymbol{x}'$ and $\boldsymbol{x}$ and $\boldsymbol{b}$ is the body force. The function $\boldsymbol{f}$ is called the pairwise response function. It has units of force per unit volume squared. All material properties are specified through the pairwise response function. The PD interaction between two points is called a bond. The integral in (1) is taken over the volume H_x, where H_x is the set of all points less than some fixed distance δ from $\boldsymbol{x}$. The distance δ is called the horizon and H_x is called the family of $\boldsymbol{x}$. (1) can be interpreted as a restatement of Newton's Second Law of Motion. In PD, only points within H_x are capable of applying a force at $\boldsymbol{x}$.

To find the form of $\boldsymbol{f}$ we begin by defining stretch as:

$$s(\boldsymbol{u}'-\boldsymbol{u},\boldsymbol{x}'-\boldsymbol{x})=\frac{\|\boldsymbol{y}'-\boldsymbol{y}\|-\|\boldsymbol{x}'-\boldsymbol{x}\|}{\|\boldsymbol{x}'-\boldsymbol{x}\|} \tag{2}$$

where $\boldsymbol{y}(\boldsymbol{x},t)=\boldsymbol{u}(\boldsymbol{x},t)+\boldsymbol{x}$ and $\boldsymbol{y}'(\boldsymbol{x},t)=\boldsymbol{u}'(\boldsymbol{x},t)+\boldsymbol{x}'$, and the double vertical bars denote the 2-norm. The coordinate $\boldsymbol{y}$ is the position of $\boldsymbol{x}$ after the deformation $\boldsymbol{u}$.

In our study, we consider a pairwise response function given by:

$$\boldsymbol{f}(\boldsymbol{u}'-\boldsymbol{u},\boldsymbol{x}'-\boldsymbol{x})=c\cdot s(\boldsymbol{u}'-\boldsymbol{u},\boldsymbol{x}'-\boldsymbol{x})\frac{\boldsymbol{y}'-\boldsymbol{y}}{\|\boldsymbol{y}'-\boldsymbol{y}\|} \tag{3}$$

where c is the bond-constant which is defined through a strain energy equivalence with classical theories. Note that the last term in the above equation is the unit vector between $\boldsymbol{x}'$ and $\boldsymbol{x}$ after deformation. For a 2D plane stress system, c is given by [2]:

$$c=\frac{12K}{\pi h\delta^3}. \tag{4}$$

K is the bulk modulus of the material in question and h is the thickness. Note that bond-based peridynamics restricts Poisson's ratio to 1/3 for a 2D plane-stress system. As a result, bond-based peridynamics has one unique material constant which [2] chooses to be the bulk modulus. The expression for the pairwise response function satisfies Newton's third law of motion in that $\boldsymbol{f}(\boldsymbol{u}'-\boldsymbol{u},\boldsymbol{x}'-\boldsymbol{x})=-\boldsymbol{f}(\boldsymbol{u}-\boldsymbol{u}',\boldsymbol{x}-\boldsymbol{x}')$. This peridynamic formulation also satisfies the balance of linear and angular momentum [2]. One can interpret (3) as the force due to the elongation of a spring.

1.1. Discretization

The domain is discretized using a square grid as shown in Fig. 1. We assume that the mass of each node is concentrated at the center of each node. As a result, the integral in (1) becomes a sum as given by (5), ignoring body forces.

$$\rho(\boldsymbol{x}_i)\ddot{\boldsymbol{u}}_i(\boldsymbol{x}_i,t)=\sum_{p=1}^{m}\boldsymbol{f}(\boldsymbol{u}_p-\boldsymbol{u}_i,\boldsymbol{x}_p-\boldsymbol{x}_i)V_p=\tilde{\boldsymbol{f}}(\boldsymbol{x}_i) \tag{5}$$

where m is the number of nodes within the horizon of node i, p is the index of a node within the horizon of node i, and V_p is the volume of node p. The horizon is chosen to be three times the mesh spacing as suggested in [3,4].

The integral over H_x becomes a sum of forces between $\boldsymbol{x}$ and each node in its family. One can view these forces as resulting from a stretched spring with spring constant c. With this interpretation, discretized PD can be seen as a nonlocal spring-lattice model. Additional information on PD can be found in [1,2]. Useful pseudocode for programming PD can be in found in [5].

1.2. Surface correction

The number of bonds between nodes near the edge of a domain is reduced since the distances between these nodes and the boundary of the domain are less than the horizon as shown in Fig. 2. The x-plane refers to a plane of constant x and the y-plane refers to a plane of constant y for point $\boldsymbol{x}$. The volumes Ω_x^+, Ω_x^- and Ω_y^+, Ω_y^- are defined as the points within the horizon of $\boldsymbol{x}$ that are divided by either the x or y-plane. Since the problem below relies on data at a surface, a correction factor must be used [6,7].

(5) now becomes:

$$\rho(\boldsymbol{x}_i)\ddot{\boldsymbol{u}}_i(\boldsymbol{x}_i,t)=\sum_{p=1}^{m}g_{ipc}\boldsymbol{f}(\boldsymbol{u}_p-\boldsymbol{u}_i,\boldsymbol{x}_p-\boldsymbol{x}_i)V_p=\tilde{\boldsymbol{f}}(\boldsymbol{x}_i) \tag{6}$$

where g_{ipc} is the correction factor for the bond between $\boldsymbol{x}_p$ and $\boldsymbol{x}_i$.

In order to find g_{ipc} we must first find the components of the force density, $g_x(\boldsymbol{x})$ and $g_y(\boldsymbol{x})$. These scalars are found by evaluating (1) for a given deformation. Note that g_x and g_y should be calculated at the location of nodes which require the