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Transformation of body force generated by non-contact sources of ultrasound in an isotropic solid of complex shape into equivalent surface stresses

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HIGHLIGHTS

- We develop a method to simulate dynamic body forces as equivalent surface stresses.
- Body forces are assumed to be confined at depths smaller than elastic wavelengths.
- The method handles arbitrary shaped surface through surface differential operators.
- Explicit formula of equivalent stress includes first and second order force moments.
- Formulas provide inputs to wave radiation models for non-contact ultrasonic sources.

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ABSTRACT

Non-contact techniques in ultrasonic nondestructive evaluation use external non-mechanical excitation (electromagnetic, heat) which interacts with the mechanical part to be tested. The part itself becomes the source of ultrasounds by transforming the non-mechanical energy into a mechanical one. This process involves the generation of dynamic body forces or of an eigenstrain that can be modeled as equivalent body forces, these forces being confined in the vicinity of the part surface. Many models developed for predicting ultrasonic field radiation in solids assume source terms given as surface distributions of stress. In order to predict ultrasonic fields radiated by non-contact sources by means of these radiation models, we developed a method to transform dynamic body forces into equivalent surface stress distributions, irrespective of the nature of the excitation. The approximate transformation relies on a second order expansion of Green's integral formulation of the elastic wave equation. To make this transformation applicable broadly, the geometry of the surface considered herein is of complex shape, implying thorough differential and tensorial analyses to achieve our aim. Some assumptions, notably isotropic elasticity, are made in deriving the transformation method, which are discussed in detail to clearly define its applicability. Numerical examples of radiated fields are given for illustration and validation.

1. Introduction

In ultrasonic Non-Destructive Evaluation (NDE), elastic waves are generated into the solid under examination by transducers. In most cases, piezoelectric transducers are used, operating from outside through a medium which mechanically

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couples them to the solid. When the use of such coupling must be avoided or if piezoelectric transducers cannot be used, alternative non-contact solutions must be implemented. For this, the part under examination is excited by an external source of a physical field of non-mechanical nature. The interaction of this field with the part converts the non-mechanical energy involved into dynamic mechanical effects within the material, which in turn becomes a source of elastic waves. Electro-Magnetic Acoustic Transducers (EMAT) [1,2] and lasers in the thermoelastic regime [3] are the two main non-contact sources currently used in Ultrasonic Testing (UT). The dynamic mechanical effects they generate are produced in the former case by electromagnetic phenomena and in the latter case by material heating. In both cases, mechanical sources are generated in a thin layer below the solid free (of coupling medium) surface. The layer thickness depends both on characteristics of the excitation fields involved and on the ability of the solid medium to transform them into dynamic mechanical energy.

It is not our purpose to describe further the underlying physical phenomena involved in these transduction processes which are still the subjects of researches in complex cases, even though the basic principles are well known and documented elsewhere [1–3]. For what follows, we only assume that this kind of transduction by non-contact sources can be modeled and that, as a result, the dynamic source of elastic waves can be written as a distribution of body forces. Note that the very nature of the mechanical excitation may be an actual body force or an eigenstrain (elastic or inelastic). In the latter case, some classical manipulations in continuum mechanics allow one to convert such strains into artificial body forces distribution. In our target applications for which the present work has been developed, namely EMAT generation of ultrasonic waves in ferromagnetic materials, both kinds of mechanical excitation coexist: the Lorentz and magnetization transduction processes are actual dynamic body forces, whereas magnetostrictive one is a pure inelastic eigenstrain, for which the conversion into artificial body force distribution has been proposed in the literature (see for example [1,4]). Regardless of its physical nature, this distribution of body forces is generally confined in the vicinity of the part surface. The notion of vicinity will be explicitly indicated by a thickness denoted by β, subsequently. In EMAT applications, β is related to the skin depth of the involved dynamic electromagnetic fields.

In ultrasonic nondestructive evaluation, accurate knowledge of the ultrasonic field radiated by the source is of paramount importance as soon as quantitative results are expected, that is to say, if the method of examination is designed to go as far as defect characterization and sizing. For this reason, in the vast literature, many models have been developed to compute field radiation by ultrasonic sources. Among them, many models were developed to predict fields radiated by mechanical sources operating at the surface of the solid. The calculation of the elastic wave field radiated by a point-source of normal or tangential stress in an elastic half-space, that is to say, the Green's function of the so-called Lamb's problem, is the subject of hundreds of papers in the literature, including exact, approximate or numerical solutions and dealing with a variety of elastic media, after Lamb's pioneering contribution [5]. For finite-size sources, a simple surface and time convolution of the time-dependent surface stress distribution with one such solution for a point source leads to predict source diffraction effects [6]. It is our aim to develop a mathematical method allowing us to use this kind of time and surface convolution models to predict the field radiated by non-contact sources.

The present work demonstrates the possibility of deriving a systematic way of transforming body forces associated to a non-contact source into equivalent sources of surface stress that can be readily used in radiation models. The basic idea is rather classical and has been used by Kawashima [7] to transform Lorentz's force generated by an EMAT into equivalent surface stress by simply integrating the force over the depth. The approximation made corresponds to a zeroth order expansion of the radiation integral of a body force distribution. The possibility to derive a higher order approximation was pioneered by Thompson [8] who derived an expression of equivalent surface stress to model magnetostriction effects produced by an EMAT in a ferromagnetic planar part. In this work, a second order expansion was worked out. However, mathematical details were not given in this paper and some final expressions were misprint. Recently, following Thompson's steps, Rouge et al. [9] published the necessary mathematical details to derive the second order expansion in terms of equivalent surface stress for a distribution of body forces at the vicinity of a planar surface. The assumption made of a planar part leads to significant simplifications (though many of them are not explicitly identified) so much so that final formulas of equivalent surface stress are rather simple.

Here, the second order expansion is derived in the case of body forces located below a free surface of complex shape. The aim is to make this transformation suitable to deal with the actual shape of parts typically tested in the industrial practice. Some tensorial and differential analyses are carried out to deal with the geometrical representation of an arbitrary surface. The present paper details the mathematical steps which allow the derivation of a first order expansion to model body force distributions as equivalent surface stresses. Then, an explicit formulation of the second order expansion is also given. It is obtained using similar mathematical calculations not fully reproduced in the paper for conciseness. The underlying assumptions and approximations made in deriving the formulas are pointed out throughout the paper to clearly discuss their applicability. The final formulation for complex shaped surfaces is readily usable. Its improved accuracy is illustrated by some numerical validations in the cylindrical case.

2. Mathematical formulation of the problem

2.1. Geometric layout definition

There are many ways to define a surface. Here, our choice, motivated by computation needs, has been made to represent the exterior surface $\partial\Omega$ of an inspected component Ω by a parametric surface description in $\mathbb{R}^3$. Thus, $\partial\Omega$ is depicted by