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Nonlinear coupled electromagnetic wave propagation: Saturable nonlinearities

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HIGHLIGHTS

- Phenomenon of nonlinear coupled electromagnetic wave propagation is considered.
- The problem is formulated with physically realistic conditions.
- An original analytic approach is used to study the problem.
- It is proved the existence of a novel (nonlinear) guided regime.

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ABSTRACT

Propagation of a sum of two monochromatic transverse electric (TE) waves in a plane dielectric layer filled with nonlinear medium is considered. Nonlinearity in the layer is described by a diagonal tensor with arbitrary functions w.r.t. squared module of the complex amplitudes of an electric field. We look for guided waves that propagate along the boundaries of the layer and decay when they move off from the boundaries. It is proved that a novel nonlinear propagation regime arises, called 'coupled TE wave.' It is shown that two TE waves – generating the coupled wave – propagate at different frequencies ω_1, ω_2 with different propagation constants γ_1, γ_2, respectively. The wave propagation problem is reduced to a nonlinear 2-parameter transmission eigenvalue problem for Maxwell's equations. An original analytical method to study the problem is suggested. For a wide class of saturable nonlinearities, it is proved the existence of isolated coupled eigenvalues (that correspond to the coupled propagation modes) and intervals of its localisation are found, zeros of the eigenfunctions are also determined. Theoretical results are illustrated with numerical calculations.

0. Introduction

The paper is devoted to studying a nonlinear interaction between two monochromatic transverse-electric (TE) waves propagating in different directions (along y and z axes, respectively, in $Oxyz$ Cartesian coordinates). The waves propagate along the boundaries of a dielectric layer filled with nonlinear medium and placed between two half-spaces with constant permittivities. The permittivity of the layer is described by a diagonal tensor $\widehat{\varepsilon} = \operatorname{diag}\{\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}\}$, where ε_{xx}, ε_{yy}, and ε_{zz} are arbitrary functions w.r.t. squared module of the complex amplitude of an electric field (i.e., we consider permittivities that describe self-action effects and are local in time and space). We look for eigenmodes that decay when they move off from the boundaries of the layer.

It is shown that these two TE waves form a new polarisation – called the 'coupled TE wave' or 'TE–TE wave' – existing only in nonlinear media (to compare, see [1–3]). It is shown that the TE–TE wave propagates at two (different) frequencies ω_1

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and ω_2 and has two real propagation constants γ_1 and γ_2. The problem is reduced to a nonlinear 2-parameter transmission eigenvalue problem for Maxwell's equations which solutions called coupled eigenvalues (or coupled propagation constants). Each pair of coupled eigenvalues corresponds to a guided TE–TE wave.

We apply an original analytical technique – called the *integral dispersion equation method* (IDEM) – to study the problem; IDEM has no connections with a bifurcation point notion or a perturbation theory. For the case of a pure monochromatic TE wave this method was developed in [4] and then successfully applied to the Kerr case [5] (the problem from [5] is actively discussed from 1980s and has been solved only recently). The main feature of the applied technique is that for saturable nonlinearities IDEM allows finding important results about coupled eigenvalues, like existence, localisation, etc., without knowing first integrals of the governing equations. IDEM can also be applied for the case of unbounded nonlinearities (e.g., Kerr or Kerr-like types); however, in this situation additional information becomes extremely important (e.g., first integrals [5]).

IDEM allows one to reduce the original 2-parameter eigenvalue problem to a system of two dispersion equations (DEs) w.r.t. the spectral parameters. It is proved that solutions to the DEs are coupled eigenvalues of the original problem and vice-versa. In fact, the system of DEs is derived for arbitrary (nonlinear) permittivities depending on squared module of the electric field. However, the existence of coupled eigenvalues is proved only with additional restrictions, in particular, the permittivities are assumed to be bounded functions that simulate saturable nonlinearities. Saturable nonlinearities are often used in nonlinear optics [6–11] in order to describe particular mechanisms of the medium permittivity in response to a strong electromagnetic field, see also Remark 5 in Section 2. Such nonlinearities are also important as they represent an alternative for the Kerr and Kerr-like nonlinearities, which are expressed through unbounded functions. However, saturable nonlinearities look more realistic from the physical standpoint than, e.g., the Kerr nonlinearity. For the case of the Kerr nonlinearity some results see in [2,3,12].

Problems of nonlinear coupled electromagnetic wave propagation [2,3,12] give interesting and important nontrivial examples of 2-parameter nonlinear eigenvalue problems with discrete sets of coupled eigenvalues, whereas in the linear theory coupled eigenvalues belong to continuous sets [13].

The paper is organised as follows: in Section 1 the full electromagnetic statement of the problem is given; in Section 2 the original problem is reduced to a nonlinear 2-parameter eigenvalue problem on a segment; in Section 3 the system of DEs is found and a theorem of equivalence is proved, zeros of the eigenfunctions are determined; in Section 4 well-known necessary results of the linear theory are presented; in Section 5 theorems of existence and localisation of coupled eigenvalues are proved; in Section 6 numerical calculations are given and compared with the theory.

1. Governing equations and statement of the problem

Let us consider a sum of two monochromatic TE waves propagating in two orthogonal directions along the boundaries of a homogeneous anisotropic nonmagnetic dielectric layer

$$\Sigma := \{(x, y, z) : 0 \leqslant x \leqslant h, -\infty < y, z < +\infty\}$$

filled with nonlinear medium. The layer is located between two half-spaces: $x < 0$ and $x > h$ in Cartesian coordinate system $Oxyz$. The half-spaces are filled with isotropic nonmagnetic media without any sources and characterised by constant permittivities $\widetilde{\varepsilon}_1$ and $\widetilde{\varepsilon}_3$, respectively. Without loss of generality we assume $\widetilde{\varepsilon}_1 \geqslant \widetilde{\varepsilon}_3$; throughout the paper we also suppose that $\widetilde{\varepsilon}_1 = \varepsilon_0\varepsilon_1 \geqslant \varepsilon_0$ and $\widetilde{\varepsilon}_3 = \varepsilon_0\varepsilon_3 \geqslant \varepsilon_0$ where $\varepsilon_0 > 0$ is the permittivity of free space. Everywhere below $\mu = \mu_0$ is the permeability of free space.

The electromagnetic field is written in the form [12]

$$\mathbf{E} = \mathbf{E}_1 e^{-i\omega_1 t} + \mathbf{E}_2 e^{-i\omega_2 t}, \qquad \mathbf{H} = \mathbf{H}_1 e^{-i\omega_1 t} + \mathbf{H}_2 e^{-i\omega_2 t},$$

where

$$\begin{aligned} &\mathbf{E}_1 = (0, E_{1y}, 0)^{\mathrm{T}}, \qquad \mathbf{E}_2 = (0, 0, E_{2z})^{\mathrm{T}}, \\ &\mathbf{H}_1 = \underbrace{(H_{1x}, 0, H_{1z})^{\mathrm{T}}}_{\text{first TE wave}}, \qquad \mathbf{H}_2 = \underbrace{(H_{2x}, H_{2y}, 0)^{\mathrm{T}}}_{\text{second TE wave}} \end{aligned}$$

are called the complex amplitudes [12,14] and we assume that

$$\begin{aligned} &E_{1y} \equiv \mathrm{E}_{1y}(x)e^{i\gamma_1 z}, \qquad H_{1x} \equiv \mathrm{H}_{1x}(x)e^{i\gamma_1 z}, \qquad E_{1z} \equiv \mathrm{E}_{1z}(x)e^{i\gamma_1 z}, \\ &E_{2z} \equiv \mathrm{E}_{2z}(x)e^{i\gamma_2 y}, \qquad H_{2x} \equiv \mathrm{H}_{2x}(x)e^{i\gamma_2 y}, \qquad H_{2y} \equiv \mathrm{H}_{2y}(x)e^{i\gamma_2 y}; \end{aligned}$$

here $\mathrm{E}_{1y}(x)$ and $\mathrm{E}_{2z}(x)$ are real-valued functions; (γ_1, γ_2) is an unknown pair of real PCs.

The permittivity inside the layer is described by a diagonal tensor $\widetilde{\boldsymbol{\varepsilon}}$. The tensor $\widetilde{\boldsymbol{\varepsilon}}$ is equal to $\varepsilon_0\boldsymbol{\varepsilon}$, where $\boldsymbol{\varepsilon} = \mathrm{diag}\,\{\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}\}$ and

$$\varepsilon_{yy} = \varepsilon_{1y} + f_1(|\mathbf{E}|^2), \qquad \varepsilon_{zz} = \varepsilon_{2z} + f_2(|\mathbf{E}|^2),$$

where f_1, f_2 are real-valued for real arguments; the form of ε_{xx} is not important as ε_{xx} does not affect the field. Until Section 5 we assume that $f_1, f_2 \in C^1[0, +\infty)$.