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Multiple scattering of elastic waves by pinned dislocation segments in a continuum

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HIGHLIGHTS

- Multiple scattering of elastic waves by dislocations is studied in perturbation theory.
- Independent scattering approximation leads to a summable series.
- A short-wavelength divergence can be renormalized.
- There are similarities with scattering by a delta function in the Schrödinger equation.

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ABSTRACT

The coherent propagation of elastic waves in a solid filled with a random distribution of pinned dislocation segments is studied to all orders in perturbation theory. It is shown that, within the independent scattering approximation, the perturbation series that generates the mass operator is a geometric series that can thus be formally summed. A divergent quantity is shown to be renormalizable to zero at low frequencies. At higher frequencies said quantity can be expressed in terms of a cut-off with dimensions of length, related to the dislocation length, and physical quantities can be computed in terms of two parameters, to be determined by experiment. The approach used in this problem is compared and contrasted with the scattering of de Broglie waves by delta-function potentials as described by the Schrödinger equation.

1. Introduction

In recent years, the interaction of an elastic wave with a dislocation has been studied within the context of continuum elasticity [1] (hereafter I). These results have been further used to obtain results, using multiple scattering theory, for the coherent propagation of elastic waves in the presence of many, randomly distributed, dislocations [2] (hereafter II) using methods that go back to the seminal works of Karal and Keller [3], Frisch [4] and Weaver [5]. Leading order perturbation theory was used to obtain formulas relating change in the speed of wave propagation to dislocation density. These formulas have been used to show that ultrasound – more specifically resonant ultrasound spectroscopy (RUS) – can be used as a quantitative probe of dislocation density in aluminum [6], with comparative advantages over X-ray diffraction (XRD) and transmission electron microscopy (TEM) [7].

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The advent of a new, quantitative, nonintrusive, tool to probe dislocation density raises hope for progress in solving long-standing challenges involving the plastic behavior of materials, such as fatigue [8], or the brittle-to-ductile [9] transition. More specifically, there are a number of applications that stand to benefit from a nonintrusive measurement of dislocation density. For example the plastic deformation in torsion of ice single crystals has been studied using hard X-ray diffraction [10] in order to ascertain the role of geometrically necessary dislocations, and size effects have been unraveled through creep measurements [11]. Also, high resolution extensometry experiments carried out on copper single crystals in tension have uncovered a rich spatiotemporal structure [12]. Distinct scales of plastic processes have been found in austenitic FeMnC steel, also using extensometry [13]. Finally, the Portevin-Le Chatelier effect [14] – in which plastic instabilities generate localized bands – seriously hampers the use of some alloys [15] and remains, to a large extent, a mystery. Recent attempts at developing a conceptual framework to understand the formation of patterns by dislocations include those of Sethna and collaborators [16], Limkumnerd and van der Giessen [17], and Rickman, Haataja and Le Sar [18].

Ultrasonic waves penetrate deep into a material and appear to be the ideal tool to probe the effects described in the previous paragraph. However, in order to probe structure, the probing wavelength must be comparable to the structure length scale. The results reported by Mujica et al. [6] are an average over a whole sample, and rely on leading order results in a long wavelength approximation of the theory [1,2]. Although these leading order results, obtained using perturbation theory in a multiple scattering framework, do provide precise and useful formulas, higher order approximations will be needed to probe shorter length scales. But, as discussed below, higher order results can diverge at high frequency because of the zero thickness of the strings used to model dislocations. This paper is devoted to the elucidation of this state of affairs.

Section 2 provides the conceptual framework and approximation scheme that will be used. Section 3 shows that a Born series for the mass operator results and that it is a geometric series that can be summed. Section 4 considers the one divergent quantity that appears in the mass operator and shows that it can be expressed in terms of a cut-off length, a quantity to be determined by experiment. Section 5 discusses differences and similarities with the related problem of scattering of de Broglie waves by delta function potentials. It is concluded that, in spite of superficial similarities, the problems differ, especially in the role played by the cut-off quantity. Section 6 has a discussion and conclusions. Technical details are spelled out in two appendices.

2. Interaction of elastic waves with dislocation segments

We shall use the notation of I and II: A homogeneous and isotropic, linearly elastic, medium of mass density ρ is described by displacements $\vec{u}(\vec{x}, t)$ away from an equilibrium position $\vec{x}$ at time t. In the absence of dislocations the dynamics is governed by the wave equation

$$\rho \frac{\partial^2 u_i}{\partial t^2} - c_{ijkl} \frac{\partial^2 u_k}{\partial x_j \partial x_l} = 0 \tag{1}$$

where the elastic constants tensor is given by $c_{ijkl} = \lambda \delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$ with (λ, μ) the Lamé constants. Longitudinal (L) and transverse (T) waves propagate with speeds $c_L \equiv \sqrt{(\lambda + 2\mu)/\rho}$ and $c_T \equiv \sqrt{\mu/\rho}$, respectively. We shall call their ratio $\gamma \equiv c_L/c_T > 1$.

A pinned dislocation segment is described by its position $\vec{X}(s, t)$ as a function of a Lagrangian parameter s and time, with orientation provided by the unit tangent $\hat{\tau} \equiv \vec{X}'/|\vec{X}'|$, where a prime denotes derivation with respect to s. We shall consider unbiased edge dislocations of length L and Burgers vector $\vec{b}$, that is, their position at equilibrium are straight lines. In the absence of external loading the dislocation dynamics is given by a linear string model

$$m\ddot{X}_k(s, t) + B\dot{X}_k(s, t) - \Gamma X_k''(s, t) = 0 \tag{2}$$

where overdots denote derivation with respect to time, and the associated boundary conditions of pinned ends are $X_k(\pm L/2, t) = 0$. In Eq. (2) the coefficient

$$m \equiv \frac{\rho b^2}{4\pi}(1 + \gamma^{-4}) \ln(\delta/\delta_0), \tag{3}$$

defines a mass per unit length (with δ and δ_0 the long- and short-distance cut-off lengths, respectively),

$$\Gamma \equiv \frac{\mu b^2}{2\pi}(1 - \gamma^{-2}) \ln(\delta/\delta_0) \tag{4}$$

is a line tension, and B is a phenomenological viscous drag coefficient [19].

When a wave propagates in the presence of dislocations, there is an interaction. The behavior of the wave in interaction with the dislocations is given, in the frequency domain, by the following equation [2] for the velocity $v_i(\vec{x}, \omega)$:

$$-\rho\omega^2 v_i(\vec{x}, \omega) - c_{ijkl} \frac{\partial^2}{\partial x_j \partial x_l} v_k(\vec{x}, \omega) = V_{ik} v_k(\vec{x}, \omega) \tag{5}$$