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Sound scattering and its cancellation by an elastic spherical shell in free space and near a free surface

E.J. Avital[a,*], T. Miloh[b]

[a] School of Engineering and Materials Science, Queen Mary University of London, 327 Mile End Road, London E1 4NS, UK

[b] School of Engineering, Tel Aviv University, Ramat Aviv, Israel

HIGHLIGHTS

- Solutions are derived for sound scattering by a flexible spherical shell.
- Shell is in free space or near a free surface.
- Shell is subject to a prescribed pressure and incoming planar sound wave.
- Exact solutions are found for a prescribed pressure eliminating scattering.
- Discrete prescribed pressure can also reduce scattering significantly.

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ABSTRACT

Sound scattering by an elastic spherical shell is analysed using linear acoustics and linear structural dynamics. It is suggested to utilize the shell's structural dynamics to reduce or even eliminate the scattered sound field, thus making it practically acoustically invisible. This can be achieved using a prescribed external pressure distribution acting on the shell's wall. Exact analytical solutions are found for that external pressure distribution, eliminating the scattered wave when the sphere is in free space or near a free surface and is subject to an incoming planar monochromatic sound wave. The latter is assumed to propagate in a direction perpendicular to the free surface (if it exists). The case of a few pressure-actuators acting on the shell's wall is also modelled and an optimal solution which reduces the sound scattering by these actuators is found. An aluminium shell of 1 m radius and 5 mm thickness, situated in fresh water is analysed for sound frequencies of up to 10 kHz. The scattered wave fields are presented as well as the external pressure distributions that eliminate these scattered sound field, i.e. achieving acoustic cloaking. Significant reduction in the scattered wave energy and the target strength of more than 10 dB are also realized using a few pressure-actuators as long as the distance between the actuators is no more than three times the incident wave length for the investigated cases.

1. Introduction

Sound scattering by a three-dimensional body is of significant importance in various engineering applications ranging from aviation to underwater acoustics. It affects the level of noise pollution that can be of environmental nuisance, the abil-

* Corresponding author. Tel.: +44 20 7882 3616.
E-mail addresses: e.avital@qmul.ac.uk (E.J. Avital), miloh@tau.ac.il (T. Miloh).

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ity to transmit information accurately and detect objects using sonar. Hence there is a significant interest in seeking ways to reduce or even eliminate sound scattering. In this study we will look at factual means of harnessing the structural dynamics of a spherical shell to reduce its scattered wave field when subject to an incoming planar sound wave.

Sound scattering by a single rigid sphere in an unbounded (free) space, is a fundamental acoustic problem that was already studied by Raleigh [1]. Both the incident planar sound wave and the scattered wave were expressed as Fourier–Legendre series. The amplitudes of the scattered wave series were found by upholding the boundary condition of zero normal velocity on the scatterer surface in the Fourier space. A similar approach can be used for a soft sphere where the boundary condition is of zero acoustic pressure, although it will result in a series that will be slower to converge than in the rigid sphere case [2]. In both cases the scattered wave field can be approximated using a monopole and a dipole located at the sphere's centre for the low frequency limit.

A comprehensive analysis for linear axisymmetric structural dynamics of a spherical shell and its effect on sound scattering and radiation in free space was given by Junger and Feit [3]. Huang and Gaunaurd [4] analysed sound scattering by an elastic spherical shell near a free surface. This means a surface of zero acoustic pressure. The surface was modelled using the image method and the collocation method was applied over the sphere's surface in order to calculate the modes' amplitudes. However, as in Avital et al. [5] who considered a plate embedded on a free surface in the Fourier–Bessel space, the collocation method yielded an ill-conditioned matrix and a least-square operation had to be used in order to produce a new better conditioned matrix.

Sound scattering by a spherical shell was also analytically and experimentally considered by Deveau and Lyons [6]. They aimed to examine the effect of the shell's material and fluid inside the shell on the acoustic target strength T_S. The latter can be defined according to [7] as;

$$T_S = 20\ \log_{10} \frac{r\ |p|_{back}}{r_{ref}\ |p|_{inc}}, \tag{1}$$

where r is the distance from the sphere's centre when measured at the direction opposite to the incident wave and $|p|_{back}$ is the scattered wave amplitude at that distance. r_{ref} is a reference distance which in this study is taken as the sphere's radius and $|p|_{inc}$ is the amplitude of the incident wave. A generalized theory of sound scattering focusing on the resonance cases of elastic spherical shells was recently presented by Mitri [8].

The present study concentrates on ways to reduce sound scattering by means of structural dynamics. The most common approach is the passive control method of applying some sort of coating over the object (or part of it) and try to match its acoustic impedance, as much as possible, to the acoustic impedance of the medium. In cases related to underwater applications, a viscoelastic coating has been commonly suggested and Patridge [9] used the approximate technique of deformed cylinder method (DCM), in order to estimate sound scattering from axisymmetric bodies partly or fully coated with a viscoelastic layer. It was shown that, as far as underwater sound propagation is considered, the specific properties of the coating layer had little effect for sound frequencies lower than 2 kHz, but at higher frequencies the target strength decreased linearly as the layer's thickness was increased. Coating using meta-materials with a negative refraction index has also been suggested. It is based on a transformation for solving the 2D wave equation [10,11]. It was suggested to use for example arrays of small cavities and channels to construct such meta-materials and this is an important active field in materials research.

Guild et al. [12] proposed building a cloaking layer around a sphere, analytically identifying two regions of scattering cancellation for electromagnetic and acoustic waves. Martin and Orris [13] suggested a hybrid approach, combining the transformational acoustic cloaking approach based on anisotropic media that is not found in nature with a wrapping layer of a more conventional isotropic material. The design was analytically optimized for cylindrical shells. Sanchis et al. [14] built a 3D cloak around a sphere using tori, experimentally achieving a reduction of 10 dB in the scattering cross section of a sphere in air for $ka \cong 2\pi$, where k is the wave number and a is the sphere's radius.

Although the idea of employing a passive sound control method seems to be attractive, it nevertheless has its own limitations. For example, Scandrett et al. [15] demonstrated both analytically and numerically that a layer of viscoelastic coating over a plate is not that effective in reducing sound radiation from the plate or sound scattering by the plate. However, adding a particular piezoelectric substance, that could change its properties through active control, points to a promising direction of sound reduction (both radiation and scattering). The potential in harnessing the structural dynamics of a shell to reduce sound scattering in an underwater environment was considered for a free surface piercing cylindrical shell by Avital and Miloh [16] and for a floating circular plate by Avital et al. [5]. An external pressure distribution acting on the shell's surface was sought in order to reduce and even eliminate the sound scattering from the object. An exact analytical pressure distribution that completely eliminated the sound scattering within the limits of the linear theory was found for the cylindrical shell, while a uniform oscillatory pressure acting on the floating plate was found capable of considerably reducing the scattered wave energy for low to mid wave-lengths (compared with the plate's diameter).

The current study can be seen as a follow up of such active control acoustic techniques, considering the fundamental case of a spherical elastic shell in an unbounded or a bounded space. Such case is of engineering importance as is evident from the short review presented in this section. It will be shown that as in the case of the cylindrical shell, there is an exact analytical solution for an external particular pressure distribution that will vibrate the shell in such a way that the scattered wave is completely cancelled in both free-space and near a free interface. The potential of using a few pressure actuators acting on the shell's surface in order to reduce the scattered wave will also be explored as it can provide a practical way of implementing such an active control method of sound-scattering reduction.