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Numerical modeling of flow evolution for an internal solitary wave propagating over a submerged ridge

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HIGHLIGHTS

- Reynolds averaged Navier–Stokes equations are adopted in the internal wave–ridge interaction.
- Transient clockwise vortex and internal hydraulic jump are significant on front slope.
- On the gentle slope, wave–ridge interaction weakens and strength of the vortex reduces significantly.
- The maximum depth is 1.78 times of incident amplitude as acute interaction.

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ABSTRACT

Numerical simulations are performed to investigate the flow evolution of a depression ISW propagating over a submerged ridge. A finite volume based Cartesian grid method is adopted to solve the Reynolds averaged Navier–Stokes equations using a $k-\varepsilon$ model for the turbulent closure. Results reveal a significant transient clockwise vortex and an internal hydraulic jump emerging on the front slope of the obstacle, during the wave–obstacle interaction. This interaction generates an asymmetrical interface and pycnocline thickness on both sides of the obstacle. During this process, the amplitude and velocity of the leading waveform increase transiently, but both quickly decrease once the wave passes the obstacle. As the ridge slope decreases, the wave–ridge interaction weakens, as well as the strength of vorticity and turbulent kinetic energy. In addition, the wave may affect the nutrient transport on the front slope to a maximum depth about 50% height of the obstacle or about 1.78 times of the incident wave amplitude when a large ISW breaks on the slope.

1. Introduction

Internal waves (IWs) or internal solitary waves (ISWs) exist at the interface of a stratified fluid. The former may compose several waves in similar amplitude, while the latter usually has a significant leading waveform with minor trailing oscillations. In the ocean, the behavior (e.g. generation, propagation and dissipation) associated with IW or an ISW has an important role in some maritime activities and ecological effect. For example, a large ISW may affect oil drilling operations [1], produces turbulent mixing [2], causes nutrient pumping [2,3], and induces fish forage [4]. In addition, the ISW once observed

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Table 1
Summary of numerical test set-up for an ISW propagating over a ridge.

Case No.	h_1 (m)	h_2 (m)	η_0 (m)	θ (°)	a_0 (m)	U_0 ($m\ s^{-1}$)	ζ [a]	ω_{max} (s^{-1})	ω_{min} (s^{-1})	K_{max} ($10^{-3}\ m^2\ s^{-2}$)	E_K ($10^{-2}\ J\ m^{-1}$)	E_P ($10^{-2}\ J\ m^{-1}$)
Ridge_01	0.1	0.4	0.05	45	0.016	0.029	0.58	29.575	−2.496	0.523	4.820	19.598
Ridge_02	0.1	0.4	0.10	45	0.032	0.044	0.66	46.685	−4.882	1.559	10.962	34.743
Ridge_03	0.1	0.4	0.15	45	0.044	0.058	0.72	65.024	−6.124	3.581	18.480	47.678
Ridge_04 [b]	0.1	0.4	0.20	45	0.056	0.068	0.78	98.860	−10.348	5.377	26.258	59.052
Ridge_05	0.1	0.4	0.25	45	0.066	0.080	0.83	102.603	−10.538	6.600	34.202	69.012
Ridge_06	0.1	0.4	0.10	26	0.032	0.044	0.66	42.697	−4.407	1.305	11.125	35.231
Ridge_07	0.1	0.4	0.20	26	0.056	0.069	0.78	85.996	−9.563	4.025	27.126	59.924
Ridge_08	0.1	0.4	0.25	26	0.066	0.080	0.83	87.507	−10.277	4.331	34.803	69.825
Ridge_09	0.1	0.4	0.10	16	0.032	0.044	0.66	34.987	−3.790	0.686	11.302	35.000
Ridge_10	0.1	0.4	0.20	16	0.056	0.069	0.78	63.897	−8.332	3.491	25.999	59.196
Ridge_11	0.1	0.4	0.25	16	0.066	0.080	0.83	64.929	−9.069	3.904	34.179	69.325
Ridge_12	0.15	0.35	0.10	45	0.036	0.036	0.93	55.624	−10.128	2.344	10.981	32.930
Ridge_13	0.15	0.35	0.20	45	0.056	0.060	1.03	64.148	−11.817	3.973	24.287	56.576

[a] $\zeta = (a_0 + h_1)/(h_1 + h_2 - h_s)$.
[b] This case was employed to verify the accuracy of the numerical model ($h_\delta = 0.04$ m; $h_s = 0.3$ m; $\rho_1 = 996\ kg\ m^{-3}$; $\rho_2 = 1030\ kg\ m^{-3}$).

with amplitude up to 170 m in the South China Sea (SCS), which had a velocity difference exceeding 3.4 m s^{-1} between the upper and lower layer could become critical for the safety of any structure in the region [5].

Over the past several decades, many researchers have conducted field observations [6,7], laboratory experiments [8–11] and numerical studies [12–15] to investigate the mechanism and process of ISW generation, propagation, breaking and energy dissipation. Among them, certain physical process associated with an ISW propagating over a submerged obstacle (e.g., run-up, run-down, breaking, turbulent mixing, reflection and transmission) have been investigated, while the wave encountering a submerged slope or irregular bottom topography. Supported by advanced field equipment, many observers have suggested that the breaking of an ISW on a sloping shelf could promote vertical mixing in the coastal oceans, but most of them have only discussed the variation in waveform, velocity and total wave energy at a fixed location, rather a spatial distribution of these parameters [16–19].

In a laboratory environment, the process of an ISW propagation and its interaction with a specific submerged obstacle have been studied in a variety of experiments [10,11,20,21]; from which it may be concluded that a depression-type ISW may produce a strong hydraulic jump with downward motion and continuous eddy diffusion as a large ISW propagating over a submerged ridge. Affected by diffusion, the leading profile of the ISW then induces a wrapped vortex on the front face of the submerged ridge, and a vortex separation at its apex. On the other hand, an elevation-type ISW yields a vortex in the lee of a submerged ridge, which resembles a surface solitary wave during its transmission process [11]. On the other hand, numerical studies have simulated the evolution of an ISW waveform and analyze the variations of wave energy as an ISW propagating over a submerged obstacle [12,14,22,23]. However, most numerical models have focused on the analysis of the temporal variations in waveform, velocity and wave energy, rather the evolution of the flow field while an ISW across a submerged obstacle. In addition, the reports on their variations are often illustrated using a series of graphical images, instead of quantified values.

Because the evolution of the flow field can hardly be obtained using nonlinear potential theory since the wave–obstacle interaction is a complex process involving highly nonlinear characteristics and shear effect. Hence, a numerical wave flume based on viscous fluid theory is necessary for investigating the problem considered in this paper. In the present study, numerical scheme that solves the Reynolds averaged Navier–Stokes (RANS) equations using a $k - \varepsilon$ model for the turbulent closure is used to investigate the flow evolution of a depression ISW over different triangular ridges in a stratified fluid. After obtaining a satisfactory verification in the numerical results with a bench mark case of a laboratory run, a series of additional numerical cases with different physical conditions are then carried out to examine the spatiotemporal evolutions of the flow field which include iso-density, streamline and vorticity during the evolution. These results could assist in understanding the evolution of the flow field as a depression ISW propagating over different triangular obstacles.

2. Formulation of problem

This study aims to simulate the flow evolution of a depression ISW propagating over different submerged triangular ridges (i.e., slope and depth ratio of upper/lower layer) in the framework of a transient two-dimensional, fully nonlinear, viscous flow. The numerical model solves the Reynolds averaged Navier–Stokes equations using a $k - \varepsilon$ model for the turbulent closure. In a 2D Cartesian frame of reference (Fig. 1), the physical domain considered is 12 m long (along x-axis) and 0.5 m deep (along z-axis), within which the fluid system has a finite pycnocline at the interface between the upper and lower layers. The physical dimensions and conditions in the experiments of Chen (2007) [11] shown in Table 1 (cases Ridge_1– 5) are adopted in all numerical cases. These include depth and density in upper and lower layers (h_1, h_2; ρ_1, ρ_2), step depth (η_0), slope of the ridge (θ), pycnocline thickness (h_δ). In the present study, the depth of the upper layer is less than that of the lower layer ($h_1 < h_2$) for producing a depression ISW propagating in the x-direction.