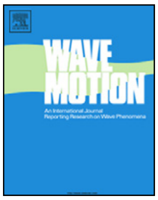
Wave Motion 54 (2015) 11–27

Contents lists available at ScienceDirect

Wave Motion

journal homepage: www.elsevier.com/locate/wavemoti



Spheroidal wave solutions for sound radiation problems in the near-field of planar structures

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HIGHLIGHTS

- We derive spheroidal wave solutions for the acoustics of planar structures.
- These frequency-dependent solutions decouple the source–field acoustical transfers.
- Their accuracy is assessed against results from a vibro-acoustic model.
- These solutions optimally represent the active and reactive power components.
- A near-field zone is defined in which the reactive component prevails.

ARTICLE INFO

Article history:
Received 6 July 2014
Received in revised form 19 November 2014
Accepted 21 November 2014
Available online 2 December 2014

Keywords:
Near-field acoustics
Radiation modes
Planar structures
Spheroidal wave functions
Reactive sound power

ABSTRACT

Current research in active noise control and in the reconstruction of vibrating sources often requires knowledge of the independent source–field components that best represent the complex acoustical transfer paths observed between a radiating structure and a given control or observation domain. In this paper, closed-form solutions are provided for the singular value expansion of the radiation operator that maps the boundary velocity of a baffled rectangular structure onto the acoustic pressure observed in the half-space domain over a hemi-spheroidal surface located at an arbitrary separation distance from the radiator, including in the near-field zone. Independent contributions of the evanescent and propagating wave components to the complex power are examined for a baffled beam when varying the frequency and the source–field distance parameter. It is shown that the reactive-to-active power ratio induced by each singular mode follows an inverse power law that scales on the product between the reduced frequency and the source–field distance parameter. A transitional region is defined in the space-frequency domain within which the reactive power components are preponderant and should be accounted for when controlling or imaging the near-field zone of a planar radiator. The optimality of the singular source modes is found to be of interest to actively reduce the active and reactive power components in the near-field zone of a radiator with a limited number of independent control channels.

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http://dx.doi.org/10.1016/j.wavemoti.2014.11.008
0165-2125/

1. Introduction

Recent developments in active noise control have focused on the performance of near-field sensing strategies using arrays of error sensors typically located within one acoustical wavelength of the primary source [1,2]. The measured near-field pressures are then governed by reactive effects associated with the evanescent field. It was found that the performance of such systems is highly dependent on a suitable definition of the near-field cost function to be minimized [1], either the mean squared pressures or the spatially-averaged normal active sound intensity. Attenuation of the radiated (also termed active) power was found to be enhanced by an optimal placement of the near-field error sensors at positions where the sound pressure reductions are the greatest during sound power minimization [2]. The near-field sensors are less prone to be contaminated by extraneous noise sources, but using in the near-field zone a pressure-based cost function strictly valid in the far-field, usually leads to an overestimate of the radiated power that is minimized [3,4]. Indeed, quadrature phase relationships occur between the pressures and the particle velocities in the near-field. They induce power cancellation effects [5] that are not accounted for in the cost function.

Active Structural Acoustic Control (ASAC) strategies have been developed in which actuators act directly on the structure and are driven to reduce the individual contributions of the first radiation modes to the active power estimated from boundary velocity or near-field pressure sensors [6–8]. The radiation modes are a set of independent source velocity distributions that best represent the sound power radiated in the far-field among all possible velocity distributions of the vibrating structure. Unlike the structural modes which may be cross-coupled by the excitation or by the acoustical fluid-loading, the radiation modes independently and optimally contribute to the radiated sound power, albeit frequency-dependent. They have been introduced by Borgiotti [9] as solutions that decouple the power radiated in the far-field by a fluid-loaded vibrating body. Equivalently, the radiation modes can be obtained from a singular value decomposition of the radiation operator [10], each of them exhibiting specific beaming properties, or from an eigen-decomposition of the active power defined on the surface of the structure [11]. Cost-efficient spatial radiation filters have been designed in ASAC experiments to estimate the contribution of the most efficient radiation modes to the power radiated by baffled planar structures from a limited number of boundary measurements [7,8]. Their frequency-dependence can be circumvented using the nesting property [12] or the use of frequency-independent weighting of the modal error signals [8]. In structural optimization, the interest focused on the least efficient radiation modes, e.g. on non-radiating surface velocity distributions, for the design of quiet structures [13]. However, when considering large-scale structures, computing the radiation modes that best contribute to the radiated sound power for a wide range of frequencies often suffers from numerical difficulties. For closed convex radiators, these issues can be circumvented using the mapped acoustic radiation modes approach, e.g. mapping the surface velocity onto a set of linearly independent patterns, the spherical harmonics of an embedded equivalent spherical source [14].

At low frequencies or for large radiating structures such as ships or facade buildings, the control domain is often in the near-field zone of the radiator. Near-field reactive effects associated with evanescent modes and the emergence of spatial coherence at sub-wavelength scales are then preponderant. Fewer works have investigated the reactive counterpart of the radiation modes. Independent surface velocity and pressure modes have been calculated, each of them decouples the complex surface acoustic power that accounts for both the reactive and active power components [15]. These singular modes occur in a one-to-one correspondence as solutions to two generalized eigenvalue problems and their relative phase difference decreases monotonically as their active-to-reactive power ratio increases. Correlations have also been established between the modal power factors of a number of radiating structures and the radiation efficiencies of their radiation modes [16].

However, there is still a need for a full analytical characterization of the near-field singular modes associated to simple geometries such as baffled planar radiators, often encountered in industrial acoustics. In particular, this would provide further insights into model-based inverse approaches such as the backward reconstruction of source velocity distributions from near-field pressure measurements. Indeed, these problems require the inversion of an ill-posed forward radiation operator [17]. A singular value decomposition of the source–field transfer matrix is then useful to filter out the least contributing singular source modes, at least those whose singular values fall below the noise threshold, in order to recover a stable and well-resolved approximation to the unknown source strength [18,19]. An exact formulation for the singular system of planar radiators would help to identify the factors that lead to an ill-conditioned transfer matrix, such as the source size and aspect ratio, the locations of the measurement positions as well as the choice of an appropriate amount of regularization.

Analytical solutions involving prolate spheroidal wave functions have been found for the radiation modes that independently contribute to the active power associated with baffled planar structures [20]. In the present work, we provide closed-form expressions for an optimal representation of the acoustical transfers that occur between a baffled planar structure that radiates in free-field and a prolate observation domain located at an arbitrary separation distance from the radiator, including in the near-field zone. In Section 2, exact expressions are derived for the singular modes of the radiation operator of a baffled beam. Section 3 builds upon these expressions to examine the link between the active and reactive power components generated by a baffled beam when varying the radiation frequency and the source–field separation distance. Section 4 proposes analytical approximations to the singular modes of the radiation operator of a baffled panel. The validity of these solutions is analysed against numerical results obtained from a vibro-acoustic model. They provide