Wave Motion 54 (2015) 170–186

Contents lists available at ScienceDirect

Wave Motion

journal homepage: www.elsevier.com/locate/wavemoti

Fast Fourier homogenization for elastic wave propagation in complex media

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HIGHLIGHTS

- We propose an FFT based homogenization method (FFH) to upscale complex elastic media.
- The FFH is based on trivial meshes, even for complex media.
- FFH method can handle both continuously or discontinuously varying fine scale models.
- The FFH can be used as a preprocess allowing simple meshes for wave equation solvers.

ARTICLE INFO

Article history:
Received 16 April 2014
Received in revised form 11 December 2014
Accepted 31 December 2014
Available online 12 January 2015

Keywords:
Elastic wave propagation
Seismology
Numerical methods
Homogenization
Upscaling
Effective media

ABSTRACT

In the context of acoustic or elastic wave propagation, the non-periodic asymptotic homogenization method allows one to determine a smooth effective medium and equations associated with the wave propagation in a given complex elastic or acoustic medium down to a given minimum wavelength. By smoothing all discontinuities and fine scales of the original medium, the homogenization technique considerably reduces meshing difficulties as well as the numerical cost associated with the wave equation solver, while producing the same waveform as for the original medium (up to the desired accuracy). Nevertheless, finding the effective medium requires one to solve the so-called "cell problem", which corresponds to an elasto-static equation with a finite set of distinct loadings. For general elastic or acoustic media, the cell problem is a large problem that has to be solved on the whole domain and its resolution implies the use of a finite element solver and a mesh of the fine scale medium. Even if solving the cell problem is simpler than solving the wave equation in the original medium (because it is time and source independent, based on simple tetrahedral meshes and embarrassingly parallel) it is still a challenge. In this work, we present an alternative method to the finite element approach for solving the cell problem. It is based on a well-known method designed by H. Moulinec and P. Suquet in 1998 in structural mechanics. This iterative technique relies on Green functions of a simple reference medium and extensively uses Fast Fourier Transforms. It is easy to implement, very efficient and relies on a simple regular gridding of the medium. Through examples we show that the method gives excellent results, even, under some conditions, for discontinuous media.

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http://dx.doi.org/10.1016/j.wavemoti.2014.12.006
0165-2125/

1. Introduction

Solving the elastic or acoustic wave equations in complex media is a difficult and a numerically expensive task, especially for media heterogeneous at scales much smaller than the minimum wavelength of the wavefield. For a given complex medium, the usual procedure to numerically model a wave propagation phenomena is first to mesh all the fine structures of the medium and then to solve the wave equation with our favorite solver. If the medium contains small scales, such a procedure is difficult and time consuming because, first, the mesh may be difficult to generate and second, the obtained fine and complex mesh induces a high numerical cost for the solver. An alternative to this simple but expensive approach is to pre-process the medium to compute an effective medium using an upscaling tool before meshing and solving the wave equation. By smoothing out all the small scales from the medium, the upscaling step makes it possible to use a sparser and simpler mesh, leading to a lower numerical cost, for the wave equation solver. In many realistic situations, the medium presents no spatial periodicity, no natural scale separation or any kind of spatial statistical invariance. This difficulty excludes most of the classical and numerical homogenization techniques to upscale the medium. We use here the non-periodic homogenization technique [1–3], which is specifically designed to upscale such general deterministic media. If the non-periodic homogenization technique is strongly inspired from the classical two scale periodic homogenization [4], it has some strong differences as it will appear later on. One of them lies in the fact that the obtained effective properties are not spatially uniform, they are just "smoother" than the original medium.

One of the important research fields in which such general media are encountered is seismology. For many applications, seismologists work with limited frequency-band data of the ground motion recorded by seismic stations. This limited frequency band can be due to attenuation or instrument response but most of the time, it is simply the seismologist himself who limits the frequency content of his data using a band-pass filter. The reason to do so is linked to limited computing power resources available to model the data, but also to a limited knowledge of the Earth's elastic structure. In the far-field of the source (an earthquake, for example), the fact that data has a maximum frequency $f_{\max}$ ensures that the wavefield has a minimum wavelength $\lambda_{\min}$. Solving the seismic forward problem using numerical methods (such as finite differences, spectral elements, etc.), that is solving the wave equation to obtain the waveform at any space location, strongly relies on this knowledge of a $\lambda_{\min}$ to accurately sample the wavefield. We assume that the elastic medium in which we need to solve the forward problem has a minimum size of characteristic heterogeneity λ_h. λ_h could be the shortest distance between two layers of a discontinuous medium or the fastest oscillation scale of a continuous medium. To estimate the scaling, as a function of $\lambda_{\min}$, of the computing time t_c necessary to solve the forward problem for a fixed signal duration, we need to distinguish two cases, depending on the regularity of the elastic medium under consideration:

1. if $\lambda_h \gg \lambda_{\min}$, we are in the smooth medium case (the wavefield oscillates much faster than the medium). In such a case, for N_s sources, the computing time t_c scales as

$$t_c \propto N_s \lambda_{\min}^{-(d+1)}, \tag{1}$$

where d is the problem dimension (2-D or 3-D). This is the optimal case in the sense that this scaling of t_c as a function of $\lambda_{\min}$ can only be improved with some extra symmetries or assumptions on the medium.

2. if $\lambda_h \ll \lambda_{\min}$, we are in the rough medium case. In such a case, for N_s sources, the computing time t_c scales as

$$t_c \propto N_s \lambda_h^{-(d+1)}. \tag{2}$$

This second case is very common in most realistic applications. In practice, this $\lambda_h^{-(d+1)}$ scaling appears differently depending on the numerical solver used to solve the wave equation. For example, if finite elements are considered, then complex, fine and discontinuous structures lead to a complex mesh which is usually difficult to generate and expensive to use. Indeed, in order to be accurate, the finite element mesh needs to honor all medium discontinuities. If finite differences are used, then small structures impose an expensive oversampling of the wavefield. The rough media case (case 2 above) is therefore a non-optimal configuration and a seismologist feels that he is paying a computing price that he should not. This intuition is linked to the fact that it is well-known from observations that, somehow, waves of $\lambda_{\min}$ wavelength are sensitive to small heterogeneity scales $\lambda_h \ll \lambda_{\min}$ only in an effective way and, if this effective medium was known, we could go back to the optimal scaling cost (case 1 above), that is a cost that scales with $\lambda_{\min}^{-4}$ and not with λ_h^{-4}. This is exactly the objective of non-periodic homogenization [1–3]: finding the upscaling operator allowing us to compute the effective medium of a given rough medium so that the numerical cost scales as $t_c \propto N_s \lambda_{\min}^{-(d+1)}$ even if $\lambda_h \ll \lambda_{\min}$. The non-periodic homogenization method gets its name by opposition to the so-called two scale periodic homogenization [4] from which it is derived, a very powerful method but limited to periodic media. A sketch summarizing the non-periodic homogenization principle in the forward modeling context is shown in Fig. 1. The main idea of the method is to compute an effective version of the original medium for which meshing and computation are simpler and cheaper without degrading the waveform accuracy. It can be seen as a pre-processing step applied to the medium before importing it into the wave equation solver. It can also be seen as a generalization of the Backus averaging (or upscaling) technique [5]. Once the homogenized medium is obtained, any wave equation solver can be used, as long as it can handle fully anisotropic and continuously varying media.

So far, we have justified the homogenization in the forward modeling context, but we could have done it for the inverse problem as well. Indeed, homogenization is very useful to build an inverse problem based on a multi-scale parameterization