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On the limit and applicability of dynamic homogenization

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HIGHLIGHTS

- Quantitative measures on the applicability of dynamic homogenization defined.
- Study of 2-phase composite and 3-phase metamaterial composite under the defined metrics.
- Study of the accuracy of homogenization in different Brillouin zones.

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ABSTRACT

Recent years have seen considerable research success in the field of dynamic homogenization which seeks to define frequency dependent effective properties for heterogeneous composites for the purpose of studying wave propagation. There is an approximation involved in replacing a heterogeneous composite with its homogenized equivalent. In this paper we propose a quantification to this approximation. We study the problem of reflection at the interface of a layered periodic composite and its dynamic homogenized equivalent. It is shown that if the homogenized parameters are to appropriately represent the layered composite in a finite setting and at a given frequency, then reflection at this special interface must be close to zero at that frequency. We show that a comprehensive homogenization scheme proposed in an earlier paper results in negligible reflection in the low frequency regime, thereby suggesting its applicability in a finite composite setting. In this paper we explicitly study a 2-phase composite and a 3-phase composite which exhibits negative effective properties over its second branch. We show that based upon the reflected energy profile of the two cases, there exist good arguments for considering the second branch of a 3-phase composite a true negative branch with negative group velocity. Through arguments of calculated reflected energy we note that infinite-domain homogenization is much more applicable to finite cases of the 3-phase composite than it is to the 2-phase composite. In fact, the applicability of dynamic homogenization extends to most of the first branch (negligible reflection) for the 3-phase composite. This is in contrast with a periodic composite without local resonance where the approximation of homogenization worsens with increasing frequency over the first branch and is demonstrably bad on the second branch. We also study the effect of the interface location on the applicability of homogenization. The results open intriguing questions regarding the effects of replacing a semi-infinite periodic composite with its Bloch-wave (infinite domain) dynamic properties on such phenomenon as negative refraction.

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1. Introduction

Recent research in the fields of metamaterials and phononic crystals has opened up intriguing possibilities for the experimental realization of such exotic phenomena as negative refraction and super-resolution [1]. It has long been understood that the involved double negative effective dynamic properties is a result of local resonances existing below the length-scale of the wavelength. This physical intuition was used to realize such materials for electromagnetic waves [2–5]. Analogous arguments and results have also been proposed for the elastodynamic case [6,7]. The central idea in this approach is that the traveling wave experiences the averaged properties of the microstructure. Therefore, it becomes imperative to define these averaged properties in a consistent manner in order to be able to explain and predict wave propagation characteristics in such materials. The theory of dynamic homogenization which seeks to define the averaged material parameters which govern electromagnetic/elastodynamic wave propagation has seen considerable research activity lately [8,9]. Subsequent efforts have led to effective property definitions which satisfy both the averaged field equations and the dispersion relation of the composite [10–16]. In addition to the complication of determining the effective dynamic properties, one must also consider the approximation involved in replacing a finite periodic composite with a homogeneous material having the homogenized dynamic properties of the composite. The approximation results from the truncating interfaces of a finite (or semi-infinite) problem but such approximations are inherently present in the negative refraction problems of metamaterial research and in transformational acoustics [17,18]. In this paper we have sought to quantify the approximation which results from replacing a semi-infinite periodic composite with what are essentially its effective dynamic properties for infinite domain Bloch-wave propagation. We show that the applicability of Bloch-wave homogenized parameters to finite problems is better over the first branch for composites with local resonance (displaying negative effective properties) than for composites without local resonance. This observation has immediate practical utility in applications which require designing for low dissipation materials with a specific impedance. More specifically, our results show that composites with localized resonance may be used in applications which require specific effective material properties in a broadband frequency range. Since composites without local resonances show a monotonic worsening of the applicability of Bloch wave homogenized properties, they do not lend themselves to similar broadband applications. It must be noted that there are different available dynamic homogenization schemes. Our study concerns one particular scheme proposed in Ref. [11] but our approach of quantifying the applicability of homogenization is general and may be applied to other schemes as well.

2. Effective dynamic properties for 1-D periodic composites

A brief overview of the effective property definitions is provided here for completeness (see Ref. [11] for details). For harmonic waves traveling in a 1-D periodic composite with a periodic unit cell Ω, the field variables (velocity, $\hat{\dot{u}}$, stress, $\hat{\sigma}$, strain, $\hat{\epsilon}$, and momentum, $\hat{p}$) take the following Bloch form:

$$\hat{F}(x,t) = F(x)\exp[i(\bar{k}x - \omega t)]. \tag{1}$$

Field equations are:

$$\frac{\partial\hat{\sigma}}{\partial x} + i\omega\hat{p} = 0; \qquad \frac{\partial\hat{\dot{u}}}{\partial x} + i\omega\hat{\epsilon} = 0. \tag{2}$$

We define the averaged field variable as:

$$\langle\hat{F}\rangle(x) = \langle F\rangle e^{iqx}; \quad \langle F\rangle = \frac{1}{\Omega}\int_{\Omega} F(x)dx \tag{3}$$

where $F(x)$ is the periodic part of $\hat{F}(x,t)$. In general, the following constitutive relations hold (see Ref. [14]):

$$\langle\sigma\rangle = \bar{C}\langle\epsilon\rangle + S_1\langle\dot{u}\rangle; \qquad \langle p\rangle = S_2\langle\epsilon\rangle + \bar{\rho}\langle\dot{u}\rangle \tag{4}$$

with nonlocal space and time parameters. For Bloch wave propagation, the above can be reduced to:

$$\langle\sigma\rangle = E^{\text{eff}}\langle\epsilon\rangle; \qquad \langle p\rangle = \rho^{\text{eff}}\langle\dot{u}\rangle. \tag{5}$$

These effective properties satisfy the averaged field equations and the dispersion relation. These definitions have been extended to the full 3-D case (Refs. [10,16,15]). As long as we are dealing with an infinitely extended composite, the frequency dependent effective properties presented above may be used to replace the composite without any approximation. In a practical situation, though, we are more interested in studying the applicability of the homogenized parameters to finite composite. This is especially true for metamaterial applications where the interesting properties of the 'negative material' arise from it being in contact with a normal positive material. Similarly, for the field of transformational acoustics/optics we are often interested in replacing finite parts of our material with periodic structures with the desired homogenized properties at the frequency of interest. We must, therefore, be able to comment upon the applicability of homogenization to finite composite, the approximation basically resulting from truncating boundaries.