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Rayleigh waves with impedance boundary conditions in anisotropic solids

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HIGHLIGHTS

- Rayleigh waves with impedance boundary conditions are considered.
- The half-space is orthotropic and monoclinic with the symmetry plane $x_3 = 0$.
- The explicit secular equations are obtained.
- They recover the secular equations of classical Rayleigh waves.

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ABSTRACT

The paper is concerned with the propagation of Rayleigh waves in an elastic half-space with impedance boundary conditions. The half-space is assumed to be orthotropic and monoclinic with the symmetry plane $x_3 = 0$. The main aim of the paper is to derive explicit secular equations of the wave. For the orthotropic case, the secular equation is obtained by employing the traditional approach. It is an irrational equation. From this equation, a new version of the secular equation for isotropic materials is derived. For the monoclinic case, the method of polarization vector is used for deriving the secular equation and it is an algebraic equation of eighth-order. When the impedance parameters vanish, this equation coincides with the secular equation of Rayleigh waves with traction-free boundary conditions.

1. Introduction

Elastic surface waves, discovered by Rayleigh [1] more than 120 years ago for compressible isotropic elastic solids, have been studied extensively and exploited in a wide range of applications in seismology, acoustics, geophysics, telecommunications industry and materials science, for example. It would not be far-fetched to say that Rayleigh's study of surface waves upon an elastic half-space has had fundamental and far-reaching effects upon modern life and many things that we take for granted today, stretching from mobile phones through to the study of earthquakes, as addressed by Adams et al. [2]. A huge number of investigations have been devoted to this topic. As written in [3], one of the biggest scientific search engines, Google Scholar returns more than a million links for request "Rayleigh waves" and almost 3 millions for "Surface waves". This data is really amazing! It shows a tremendous scale of scientific and industrial interests in this area.

For Rayleigh waves their explicit secular equations are important in practical applications. They can be used for solving the direct (forward) problems: evaluating the dependence of the wave velocity on material parameters, especially for solving

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the inverse problems: to determine material parameters from measured values of wave velocity. Therefore, explicit secular equations are always the main purpose for any investigation of Rayleigh waves.

In the context of Rayleigh waves, it is almost always assumed that the half-spaces are free of traction. As mentioned in [4], in many fields of physics such as acoustics and electromagnetism, it is common to use impedance boundary conditions, that is, when a linear combination of the unknown function and their derivatives is prescribed on the boundary. In the other hand, when studying the propagation of Rayleigh waves in a half-space coated by a thin layer, the researchers often replace the effect of the thin layer on the half-space by the effective boundary conditions on the surface of the half-space (see, for examples, Achenbach and Keshava [5], Tiersten [6], Bovik [7], Steigmann and Ogden [8], Vinh and Linh [9,10], Vinh and Anh [11], and Vinh et al. [12]). These conditions lead to the impedance-like boundary conditions on the surface. As addressed in [13,14], a thin layer on a half-space is a model finding a broad range of applications, including: the Earth's crust in seismology, the foundation/soil interaction in geotechnical engineering, thermal barrier coatings, tissue structures in biomechanics, coated solids in materials science, and micro-electro-mechanical systems. Rayleigh waves with impedance boundary conditions are therefore significant in many fields of science and technology. However, very few investigations on Rayleigh waves with impedance boundary conditions have been done. In [15] Malischewsky considered the propagation of Rayleigh waves with Tiersten's impedance boundary conditions and provided a secular equation. Recently, Godoy et al. [4] investigated the existence and uniqueness of Rayleigh waves with impedance boundary conditions which are a special case of Tiersten's impedance boundary conditions. In works [4,15] the half-space is assumed to be isotropic. Nowadays, anisotropic materials are widely used in various fields of modern technology. The investigations of Rayleigh waves with impedance boundary conditions in anisotropic solids are therefore significant in practical applications.

The main purpose of this paper is to study the propagation of Rayleigh waves with Tiersten's impedance boundary conditions [15] in anisotropic elastic half-spaces. Two cases of anisotropy are considered: orthotropic materials and monoclinic ones with the symmetry plane $x_3 = 0$ (see [16]). For the orthotropic case the secular equation is obtained by employing the traditional approach. It is an irrational equation and it provides a new version of the secular equation for isotropic materials. For the monoclinic case, for obtaining the secular equation we use the method of polarization vector and the secular equation obtained is an algebraic equation of eighth-order. When the impedance parameters vanish, this equation coincides with the secular equation of Rayleigh waves with traction-free boundary conditions.

2. Orthotropic half-spaces

Consider an elastic half-space which occupies the domain $x_2 \geq 0$. We are interested in the plane strain such that:

$$u_i = u_i(x_1, x_2, t), \quad i = 1, 2,\ u_3 \equiv 0 \tag{1}$$

where t is the time. Suppose that the half-space is made of compressible orthotropic elastic material, then the strain–stress relations are [16]:

$$\begin{cases} \sigma_{11} = c_{11}u_{1,1} + c_{12}u_{2,2} \\ \sigma_{22} = c_{12}u_{1,1} + c_{22}u_{2,2} \\ \sigma_{12} = c_{66}(u_{1,2} + u_{2,1}) \end{cases} \tag{2}$$

where σ_{ij} and c_{ij} are respectively the stresses and the material constants, commas indicate differentiation with respect to spatial variables x_k. The elastic constants $c_{11}, c_{22}, c_{12}, c_{66}$ satisfy the inequalities:

$$c_{ii} > 0, \quad i = 1, 2, 6, \qquad c_{11}c_{22} - c_{12}^2 > 0 \tag{3}$$

which are necessary and sufficient conditions for the strain energy to be positive definite. In the absence of body forces, equations of motion are:

$$\begin{cases} \sigma_{11,1} + \sigma_{12,2} = \rho\,\ddot{u}_1 \\ \sigma_{12,1} + \sigma_{22,2} = \rho\,\ddot{u}_2 \end{cases} \tag{4}$$

where ρ is the mass density, a superposed dot signifies differentiation with respect to t. Introducing (2) into (4) leads to the equations governing infinitesimal motion, expressed in terms of the displacement components, namely:

$$\begin{aligned} &c_{11}u_{1,11} + c_{66}u_{1,22} + (c_{12} + c_{66})u_{2,12} = \rho\ddot{u}_1 \\ &c_{66}u_{2,11} + c_{22}u_{2,22} + (c_{12} + c_{66})u_{1,12} = \rho\ddot{u}_2. \end{aligned} \tag{5}$$

Now we consider the propagation of a Rayleigh wave, travelling with velocity $c\,(>\ 0)$ and wave number $k\,(>\ 0)$ in the x_1-direction and decaying in the x_2-direction, i.e.:

$$u_i \to 0 \quad (i = 1, 2) \text{ as } x_2 \to +\infty. \tag{6}$$

Suppose that the surface $x_2 = 0$ is subjected to impedance boundary conditions such that [4,15]:

$$\sigma_{12} + \omega Z_1 u_1 = 0, \qquad \sigma_{22} + \omega Z_2 u_2 = 0 \quad \text{at } x_2 = 0 \tag{7}$$