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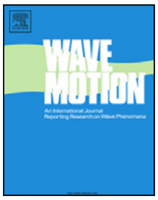

Numerical simulations of the primary Bjerknes force experienced by bubbles in a standing ultrasonic field: Nonlinear vs. linear

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HIGHLIGHTS

- We perform simulations of the primary Bjerknes force in a nonlinear standing wave.
- We examine the translation of the bubbles in the cavity at high amplitudes.
- We analyze the importance of both nonlinearities (bubble dynamics, acoustic wave).
- These nonlinearities strongly affect the force field and the bubble motion.
- We also model the nonlinear ultrasonic field after the bubbles form agglomerates.

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ABSTRACT

The primary Bjerknes force experienced by a population of multiple bubbles in a liquid set in a nonlinear ultrasonic standing field and their translation are calculated and analyzed by numerical simulations. The force field is evaluated by considering the nonlinear bubble oscillations as well as the nonlinear character of the ultrasonic pressure field (both variables are unknown in the coupled nonlinear differential system). The results at small amplitudes agree with the classical theory on bubble translation, depending on the driving frequency in relation to the bubble resonance. It is shown that, when amplitudes are raised, the force field exhibits important modifications that strongly affect the motion of the bubbles and the way they form agglomerates. An analysis is performed on the importance of the terms in the differential system that provoke (a) the nonlinearity of the bubble oscillations and (b) the nonlinearity of the acoustic wave. This study reveals that both features should be considered to better approximate the primary Bjerknes force field. Simulations of the nonlinear ultrasonic field after the bubbles form agglomerates under the influence of this force are also performed.

1. Introduction

In this paper, we investigate the forces experienced by a population of bubbles in a liquid under the effect of a nonlinear ultrasonic standing wave (the primary Bjerknes force). The primary Bjerknes force is the net radiation force acting on a bubble in a standing-wave ultrasonic field [1,2], as opposed to the secondary Bjerknes force caused by the wave radiated

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by a bubble on the other surrounding bubbles [3]. The primary Bjerknes force results from the pressure gradient across the oscillating bubble. Our goal here is to analyze the effects of nonlinearities (bubble oscillations, acoustic wave) that appear when the pressure amplitude increases on the primary Bjerknes force field and how these effects modify the motion of a population of bubbles. Similar studies have already been performed [4,5] but, as explained later, from a different perspective. They consider other factors, such as the equilibrium of a single bubble, and they use a different bubble model and consider a different theoretical acoustic wave (a given linear wave). In this paper, we consider not only nonlinearly oscillating bubbles but a nonlinear ultrasonic standing wave as well (for us, the acoustic pressure is an unknown variable of the differential problem). Moreover, we consider a population of bubbles, not only a single bubble. However, our model does not consider the collapse of bubbles.

Bjerknes forces are of interest in many industrial and medical applications for which they have a significant importance [6,7]: the process of micro-bubble froth separation [8], electrodeposition of metals using acoustically excited gas bubbles [9], cleaning applications of solid surfaces by acoustically driven bubbles [10], biomedical implications of cavitation bubbles in vivo [11] and in vitro [12–14], lab-on-a-chip devices applied to particle manipulation using oscillating microbubbles [15,16], and cavitation bubbles in sonochemistry [17,18].

A complete review on Bjerknes forces can be found in the recently published papers by Galavani et al., Xi et al., and Louisnard [19–21]. Ellenberger et al. and Fan et al. studied the experimental manifestation of the primary Bjerknes force in a bubble column [6,7].

Experimental studies have shown that the behavior of oscillating bubbles in a standing ultrasonic field becomes nonlinear at high amplitudes (see, for example, [22]). Therefore, the linear description of the primary Bjerknes force becomes inadequate at high amplitudes, and a nonlinear description is required.

Several theoretical studies on the nonlinear description of the force have supported the experimental findings [4,5,19,21,23,24]. Such studies are usually performed for sinusoidal acoustic fields (linear standing acoustic wave) and a single collapsing bubble (nonlinear bubble oscillations). Galavani et al. analyzed the effect of a linear standing wave in a resonator on a single bubble by considering the collapse of the bubble [19]. A study on the motion of a single bubble with a nonlinear shape response in a linear standing wave was presented in Ref. [23]. Koch et al. studied the effect of a linear acoustic field with varying amplitude and phase on a single bubble by taking into account the primary and secondary Bjerknes forces [24]. The main conclusion of these studies is that the linear description of the primary Bjerknes force is inadequate and that the nonlinearity of bubble oscillations must be taken into account to better approximate the primary Bjerknes force field and describe the behavior of the bubbles. Louisnard recently published a model for the study of complex bubble structures in a cavitating fluid, including the primary Bjerknes force [21]. In Ref. [4], the effect of the primary Bjerknes force on a single bubble is studied for high pressure amplitudes by considering a linear acoustic field but nonlinear bubble oscillations. The authors find that the bubble is repelled from the pressure antinode at high amplitude and that the bubble linear theory (harmonic bubble oscillations) cannot predict this result. In Ref. [5], the authors study the equilibrium position of a single bubble in a vertical column of water under the effect of the primary Bjerknes force and buoyancy as a function of the pressure amplitude by considering nonlinear bubble oscillations due to a linear acoustic field. They find that the equilibrium position is shifted away from the pressure antinode when pressures increase and that the linear theory cannot predict this result. They also study this problem experimentally, and qualitative agreement is obtained.

Our objective here is to analyze the primary Bjerknes force acting on nonlinearly oscillating bubbles under the effect of a nonlinear standing acoustic wave.

Although the bubble equation used in Refs. [4,5] (the Keller equation written in bubble radius variation) and the one we use in this paper (the Rayleigh–Plesset equation written in bubble volume variation) are based on the nonlinear bubble dynamics described in the Rayleigh–Plesset approximation and they both have many common hypotheses (spherical bubble, spatially uniform conditions within the bubble, no body force considered, constant gas content of the bubble, vapor pressure negligible, thermal damping neglected, adiabatic changes in the gas), we apply several additional restrictions to the equation:

(i) Our model neglects the radiative damping of the bubble (the liquid is assumed to be incompressible in the derivation of the bubble equation), whereas in Refs. [4,5], the loss of energy by sound radiation from the bubble is considered.
(ii) Our model neglects the surface tension of the bubble, whereas in Refs. [4,5], the surface tension of the bubble is considered.
(iii) Our model is limited to moderate bubble oscillations (due to the second-order Taylor expansion of the adiabatic law used in the derivation of the bubble equation in volume variation), and the bubble implosion cannot be modeled, whereas no such approximation is made in the derivation of the bubble equation used in Refs. [4,5], for which the entire adiabatic law is considered without restriction to the second order, and the bubble collapse can be modeled.

There are important differences between the models of Refs. [4,5] and the one used in this paper to study the effect of the primary Bjerknes force on the nonlinear oscillating bubbles at several pressure amplitudes:

(i) We consider an acoustic signal that is modified by the nonlinear effect of the oscillating bubbles and is becoming nonlinear (the acoustic field is thus non-harmonic and contains many frequencies; and the acoustic variable is an unknown variable in the differential system that must be determined during the calculations, like the bubble oscillations), as opposed to Refs. [4,5], for which the authors consider a sinusoidal acoustic pressure (which is a known and given field).