Wave Motion 51 (2014) 1349–1363

Contents lists available at ScienceDirect

Wave Motion

journal homepage: www.elsevier.com/locate/wavemoti

Lamb wave scattering by a symmetric pair of surface-breaking cracks in a plate

Andrew Golato [a], Ramazan Demirli [b], Sridhar Santhanam [a,*]

[a] *Mechanical Engineering Department, Villanova University, Villanova, PA 19085, USA*

[b] *Center for Advanced Communications, Villanova University, Villanova, PA 19085, USA*

HIGHLIGHTS

- The scattering of Lamb waves by a symmetric crack pair is studied.
- Part through cracks reflect and transmit Lamb and SH wave modes.
- Energy coefficients are a function of incidence angles, and crack depths.
- Support for the computed results is provided by experiments.

ARTICLE INFO

Article history:
Received 7 February 2014
Received in revised form 8 June 2014
Accepted 5 August 2014
Available online 14 August 2014

Keywords:
Symmetric-cracks
Energy-coefficients
Lamb
Oblique

ABSTRACT

The scattering problem of a Lamb wave incident on a symmetric pair of surface-breaking transverse cracks in a plate is considered. The Lamb wave is assumed to be obliquely incident on the crack plane. Since the cracks are part-through, the scattered field will contain reflected as well as transmitted waves. The energy of the incoming wave is partitioned into reflected and transmitted wave modes. Energy coefficients of the reflected and transmitted waves are calculated as a function of incident frequency and crack depth. The incidence angle of the incoming wave is also treated as a parameter. Both the reflected and transmitted wave fields are considered as linear superpositions of all real and complex wave modes in the plate. Decomposition of modes is achieved with the help of an orthogonality condition based on the principle of reciprocal work. Continuity of displacement and stress fields is imposed at the crack plane. Energy coefficients for reflection and transmission are obtained from the mode amplitudes. Energy coefficients are shown to be a strong function of incident frequency and crack depth. Experiments are conducted with a PZT transducer network interacting with a symmetric pair of machined cracks in an aluminum plate. Trends predicted by the analysis are reflected in the experimental results.

1. Introduction

The use of guided Lamb waves for structural health monitoring of thin two-dimensional structures has been the subject of much investigation in the recent past [1–5]. Lamb waves can travel large distances without significant attenuation. Since Lamb waves are scattered in measurable ways by defects, they are very suitable for defect detection, identification, and location [6–9]. Edge waves can also arise in thin elastic structures and can be used for non-destructive evaluation and measurement of certain material properties [10–12].

* Corresponding author. Tel.: +1 6105197924.
E-mail address: sridhar.santhanam@villanova.edu (S. Santhanam).

http://dx.doi.org/10.1016/j.wavemoti.2014.08.002

In order for Lamb waves to detect cracks and crack-like defects in plates, it is important to characterize the scattering of these waves by cracks. This subject has been well explored in the literature. One of the early studies by Rokhlin [13] analyzed the scattering of guided waves by a crack situated on the plane of symmetry of an elastic layer. In a related problem, Angel and Achenbach [14] tackled the scattering of Rayleigh waves as they interact with a surface breaking crack. Alleyne and Cawley [15] used finite element analysis to examine the interaction of Lamb waves with crack-like defects. More recently, Kim and Roh [16] examined the problem of guided wave scattering by a two-dimensional rectangular notch. Reflection and transmission coefficients were determined as a function of notch dimensions. Benmeddour et al. [17] considered a similar problem with a plate containing a symmetrical notch. Analytical and finite element methods were used to determine scattering coefficients of Lamb modes. Castaings et al. [18] considered a problem of vertical cracks in a plate interacting with incident fundamental guided wave Lamb modes. The method of mode decomposition was used to calculate scattering coefficients for the problem of Lamb waves incident normal to the crack plane. Flores-Lopez and Gregory [19] used the projection method to solve for scattering coefficients of Lamb waves scattered by a transverse surface breaking crack in a plate under normal incidence conditions. Cho and Rose [20] and Wang et al. [21] used the boundary element method to examine similar problems.

While the problem of scattering of normally incident Lamb wave modes on vertical cracks in a plate has been well studied, the related problem of oblique incidence has received little attention. When a Lamb wave is obliquely incident on a vertical crack in a plate, the scattering phenomena are expected to be different and more complex than the case of normal incidence. This is indicated by the results that have been obtained from the related problem of Lamb waves striking a plate edge at an oblique angle and the scattering caused by the interaction with the edge. For example, Santhanam and Demirli [22] have recently calculated reflection coefficients of Lamb waves for the oblique incidence problem on a plate-edge. Additional SH (Shear Horizontal) and symmetric Lamb modes are scattered by the plate edge when the incident wave is a fundamental symmetric mode incident at an oblique angle. In another recent study, Wilcox et al. [23] used the finite element method to analyze the problem of scattering of Lamb waves incident on an infinitely long stiffener.

In this work, we present an analysis of the problem of scattering of fundamental Lamb wave modes (A_0 and S_0) that are incident at an oblique angle on a symmetrical, vertical, surface-breaking crack geometry in a plate. The reflected and transmitted wave fields from the crack are expressed as a linear superposition of all possible guided wave modes of an infinite plate. The amplitude coefficients of the scattered wave fields are obtained by enforcing stress and displacement boundary and continuity conditions at the plane of the crack. An orthogonality condition obtained from the principle of reciprocal work is utilized in the process. Scattering energy coefficients are calculated as a function of the principal parameters of the problem: incident frequency, crack depth, and incidence angle. Experiments were conducted to examine whether the trends predicted by the analysis could be reproduced. A set of PZT (Lead Zirconate Titanate) transducers were attached to an aluminum plate containing a symmetric machined crack. The amplitudes of Lamb waves transmitted across the crack plane were measured.

This work significantly extends earlier work by the authors [22,24]. In [22], the authors considered the reflection of Lamb waves from a plate edge. The problem considered in this work is more involved in that it deals with both reflection and transmission from a symmetrically cracked section in a plate. Some preliminary and very limited results for the symmetrical crack problem, based on a different analytical technique (collocation) have been presented in [24].

2. Analysis

The geometry studied is that of an infinite, isotropic, homogeneous, linear elastic plate of thickness $2h$ (Fig. 1). The upper and lower surfaces of the plate ($x_2 = -h$ and $x_2 = +h$) are traction free. The plate contains a pair of surface-breaking cracks, each of infinite length (in the x_3 direction) and of finite depth a in the x_2 direction as seen in Fig. 1. A plane Lamb wave mode (L_{inc}) is incident at an oblique incidence angle of θ_{inc}, with respect to $+x_1$, on the x_2–x_3 plane of the symmetric crack pair. The plate is of thickness $2h$, with each surface breaking crack of depth a, creating a cracked section ratio of $2a/2h$.

The incident Lamb wave (L_{inc}) is assumed to be either the fundamental A_0 or S_0 mode with a circular frequency of ω. Depending on the frequency ω, multiple reflected and transmitted waves can be produced. The scattered wave-fields can be expressed as a linear superposition of all permissible eigenmodes of a plate [22,25,26] of infinite extent. Fig. 1 shows some of these eigenmodes ($L_{t1}, L_{t2}, L_{tn}, L_{r1}, L_{r2}, \ldots$), each with a different transmitted or reflected angle. The energy coefficients of these scattered modes are dependent on the angle of incidence, θ_{inc}, the nature of the incident mode (A_0 or S_0), frequency ($\omega = 2\pi f$) of the incident fundamental mode, and the crack depth ratio ($2a/2h$).

2.1. Guided wave modes: A brief overview

The theory of guided waves propagating in a plate with traction free surfaces is well known [27]. For free waves propagating in a plate with stress-free surfaces, the wave equation yields the well-known dispersion equations which relate the wave-number k of the wave to the circular frequency ω and material properties (bulk longitudinal wave speed c_p and shear wave speed c_s). Multiple solutions arise from the dispersion equations in the form of symmetric and anti-symmetric guided wave modes called Lamb modes [26]. Shear Horizontal (SH) modes also can arise as symmetric and anti-symmetric waves. The wave number k for all these modes are found from the dispersion equations, which are well documented in the literature [26,27] and will not be reproduced here. The results from an analysis of the dispersion equations are expressed as dispersion curves which are plots of the phase speed ($=\omega/k$) against the frequency ω. Dispersion curves for steel can