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Nonlocal Maxwellian theory of sound propagation in fluid-saturated rigid-framed porous media

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HIGHLIGHTS

- We propose a macroscopic nonlocal theory of sound propagation in rigid-framed porous media saturated with a viscothermal fluid.
- This theory takes not only temporal dispersion into account, but also spatial dispersion.
- An alternative procedure for homogenization is expressed, taking advantage of an acoustics–electromagnetics analogy.
- No explicit scale separation of type asymptotic approach is required to perform the upscaling procedure.

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ABSTRACT

Following a deep electromagnetic–acoustic analogy and making use of an overlooked thermodynamic concept of acoustic part of the energy current density, which respectively shed light on the limitations of the near-equilibrium fluid-mechanics equations and the still elusive thermodynamics of electromagnetic fields in matter, we develop a new nonperturbative theory of longitudinal macroscopic acoustic wave propagation allowing for both temporal *and spatial* dispersion. In this manner, a definitive answer is supplied to the long-standing theoretical question of how the microgeometries of fluid-saturated rigid-framed porous materials determine the macroscopic acoustic properties of the latters, within Navier–Stokes–Fourier linear physics.

1. Introduction

The recent focus on metamaterials in electromagnetics and acoustics – materials whose response is crucially determined by the geometrical arrangement of their constituents – has highlighted the intricate nature of the relationship that exists, in general, between microgeometry and macroscopic wave properties. A precise understanding, in the full range of geometries, would be highly desirable in view of the possible implementation of novel concepts and ideas appearing in a growing metamaterial literature. In particular, for the class of fluid-saturated porous materials with motionless solid frame and connected fluid phase being the foundation for acoustic analyses,[1] the importance of a theoretical clarification of the general geometry/acoustics relationship must be evident to all, as it determines in part our power of designing materials of all kind, exhibiting desired acoustical properties.

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[1] The solid being motionless either because of large mass, rigidity, or both.

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The purpose of the present paper is to provide a theoretical nonlocal framework allowing such clarification to be made, for the propagation of macroscopic longitudinal waves, assuming for simplicity macroscopic homogeneity and isotropy, or otherwise, propagation along a symmetry axis.[2]

The traditional approach to this problem assumes long-wavelengths, *i.e.*, large scale separation between typical macroscopic wavelengths and typical pore sizes. Relying upon the two-scale asymptotic homogenization method [1–8], it leads to considering that, to the leading order, the media react *locally* and can be described in terms of a frequency-dependent effective density accounting for inertial and viscous effects, and a frequency-dependent effective compressibility accounting for thermal effects [9–12]: spatial dispersion only appear as a second order approximation, meaningful when the wavelengths reduce sufficiently. In the literature the corresponding theory is often referred to as the equivalent-fluid theory (see, *e.g.*, [12]). It is generally found very successful. Although, this traditional homogenization approach also leads to Biot's theory when generalized to allow for the frame elasticity [13,8], and consequently describes in a satisfactory manner the acoustic properties of many more porous materials, multilayered or not, utilized in noise control [14], it nevertheless expresses a *perturbative* approach of spatial dispersion phenomena, which cannot provide the physical solution in the full range of geometries. Despite its undeniable success and wide acceptance, it is missing an important part of the nonlocal wave physics. The inclusion of frame rigidity is beyond the scope of the present study, and not relevant for the main results. Keeping working with rigid-framed materials, the following general considerations make apparent limitations, of physical nature, of the two-scale asymptotic homogenization method.

Comparing the structure of the macroscopic equivalent-fluid theory with that of the macroscopic electromagnetic theory, it appears that the effective density may be viewed as the acoustic counterpart of the effective electric permittivity, whereas the effective compressibility may be viewed as the acoustic counterpart of the effective magnetic permittivity. Now, recent studies on electromagnetic metamaterials have shown that for suitably microstructured materials, spatial dispersion effects are not necessarily small corrections meaningful in the short-wavelength limit: strong spatial dispersion effects at long-wavelengths can be found [15], implying that the permittivities should be considered, in general, as nonlocal operators described in terms of wavenumber-dependent as well as frequency-dependent kernels. Thus, the time-variable and spatially-variable polarizations responses induced by the presence of the waves, appear to depend not only on the temporal variations of the fields, but also – and in essential manner for some microstructures – on their spatial variations. The novel metamaterial-type of behaviors appear intimately related to the latter dependences, which may lead to strong effects even at long wavelengths.

The same should be true in acoustics: the effective density and compressibility should be viewed in general as nonlocal operators described in terms of wavenumber-dependent as well as frequency-dependent kernels. In fact, for materials with embedded structures in the form of Helmholtz resonators, resonant behaviors can appear at long-wavelengths [16], which are missed by the classical equivalent-fluid: at long wavelengths and in first approximation, this description always represents the density and compressibility functions in terms of *distributions of relaxation times*—purely damped terms, see *e.g.* [17] or [10, Appendix A] and [11, Appendix C], which cannot produce any Helmholtz's resonant behavior. The origin of this failure is that long-wavelength resonances are nothing but strong long-wavelength spatial dispersion effects—absent by definition in the classical two-scale homogenization approach.

To see that the resonances are closely related to spatial dispersion, it suffices to observe that the fluid is not going in and out of a resonator, without simultaneously producing by mass conservation, spatial variations in the macroscopic wavefield. But spatial dispersion, by definition, expresses the dependence of the macroscopic properties of matter on the spatial variations of the fields [18, p. 360], hence the necessary connection between resonances and spatial dispersion.

The new nonperturbative physical theory we develop here, does not rely on asymptotic homogenization techniques, by nature not suitable to address the full physics of our wave propagation problem. We follow instead, as our heuristic guide, a deep formal analogy postulated to exist between the structure of macroscopic electromagnetic wave propagation theory on one hand, and the structure of the wanted macroscopic acoustic theory on the other hand. This analogy, which highlights both the limits of our current understanding of the fluid-mechanics equations, and of the thermodynamics of electromagnetic fields in matter, will accordingly not be written in full; nevertheless, within Navier–Stokes–Fourier physics it will successfully lead us to the definition of the wanted new, fully nonlocal, macroscopic acoustic theory of longitudinal acoustic waves in fluid-saturated rigid-framed porous materials.

Once the general concepts of temporal and spatial dispersion are borrowed from electromagnetics, the question is to distribute the right – temporal and spatial dispersion – effects in the right acoustic 'polarizations'; one resembling electric polarization and corresponding to the 'density' permittivity, the other resembling magnetization and corresponding to the 'compressibility' permittivity, the distribution being nonunique. The main idea of the nonperturbative homogenization performed in this paper is that, when the distribution is performed in the right physical manner, permittivities are obtained which are independently determinable by means of simple action–response problems. It turns out that the right physical way of doing the distribution, and working out the solutions of the action–response problems, is the expression of an overlooked fundamental *thermodynamic concept* henceforth referred to as *the acoustic part of the energy current density*. Within Navier–Stokes–Fourier longitudinal wave physics we shall see that a usable expression exists for this concept [19]. In the absence of a similar thermodynamic concept of an 'electromagnetic part of the energy current density', the method

2 These material symmetries could be removed without changing the general principles of the present nonlocal physical framework of solution.