



# Regular wave integral approach to the prediction of hydrodynamic performance of submerged spheroid



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## HIGHLIGHTS

- A complete regular wave integral technique is developed.
- Free surface Green function is approximated by a harmonic function expansion.
- Hydrodynamic performance of a submerged spheroid is examined in the expansion scheme.
- The present method is applicable for general underwater body flow.
- The thin body assumption in author's previous work is removed.

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## ABSTRACT

A free surface Green function method is employed in numerical simulations of hydrodynamic performance of a submerged spheroid in a fluid of infinite depth. The free surface Green function consists of the Rankine source potential and a singular wave integral. The singularity of the wave integral is removed with the use of the Havelock regular wave integral. The finite boundary element method is applied in the discretisation of the fluid motion problem so that the panel integral of the Rankine source potential is evaluated by the Hess–Smith formula and the panel integral of the regular wave integral is evaluated in a straightforward way due to the regularity nature. Present method's results are in good agreement with earlier numerical results.

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## 1. Introduction

For a free surface source submerged in a uniform stream, the velocity potential of the flow around the source is expressed as

$$\phi = \frac{\epsilon}{4\pi} G$$

for source strength  $\epsilon$  and translating free surface Green function  $G$ . If the source is replaced by a submerged body, a surface point of the body can be identical to a source point and the velocity potential around the body becomes the integral of the potentials of the point sources distributed on the body surface  $S$ :

$$\phi = \int_S \frac{\epsilon}{4\pi} G ds. \quad (1)$$

Thus it remains to find the unknown strengths of point sources on the body surface. To do so, it is necessary to evaluate the Green function, which consists of Rankine source potential, its image with respect to calm water surface and singular wave

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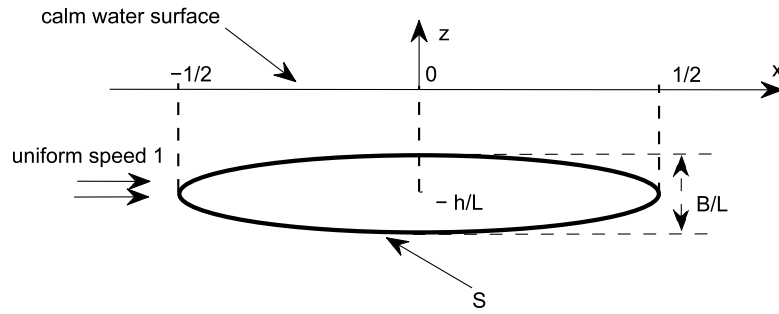


Fig. 1. Profile of the fluid motion model in the non-dimensional coordinate frame projected on the central plane  $y = 0$ .

integral presented by Havelock [1,2]:

$$G = -\frac{1}{|(x, y, z) - (\xi, \eta, \zeta)|} + \frac{1}{|(x, y, -z) - (\xi, \eta, \zeta)|} - K, \tag{2}$$

where the singular wave integral  $K = \lim_{\mu \rightarrow 0+} K^\mu$  is a limit of the regular wave integral

$$K^\mu = \frac{4\nu}{\pi} \operatorname{Re} \int_0^{\pi/2} d\theta \int_0^\infty \frac{e^{k[z+\zeta+i(x-\xi)\cos\theta]} \cos[k(y-\eta)\sin\theta]}{k \cos^2\theta - \nu - i\mu \cos\theta} dk \tag{3}$$

with respect to dissipation number  $\mu$ . Here  $\nu$  is wave number,  $(\xi, \eta, \zeta)$  is a source point and  $(x, y, z)$  is a field point.

Traditionally, the singular wave integral  $K$  is decomposed as its Cauchy principal value, which is in a double integral form, and a single integral defined by a half circle integral in the vicinity of the singularity  $\nu$  (see [3]). Then both the double and single integrals are evaluated in elementary function expansions (see [4–9]).

Recently, a regular wave integral method was developed by the author [10] from [11] to simulate free surface wave caused by a thin spheroid submerged in a uniform stream. The singular wave integral  $\partial_x K$  is approximated by the regular wave integral  $\partial_x K^\mu$  for small  $\mu$  and  $\partial_x K^\mu$  is integrated straightforwardly to produce a harmonic function expansion. By using Michell thin body formulation [12], the strength  $\epsilon$  in (1) can be approximated by body surface geometry function and body surface can be projected into the vertical plane  $y = 0$ . Since the surface wave is defined by the velocity component  $\partial_x \phi$ , the partial derivative  $\partial_x$  can be cancelled by the integral on the projected surface on the  $Oxz$  plane and thus free surface wave is evaluated in a form of harmonic function expansion.

The purpose of the present paper is to present a simple harmonic function expansion of the regular wave integral  $K^\mu$  and to employ the expansion to solve completely the potential flow around a submerged spheroid in a uniform stream without the thin body assumption. The present method is based on the boundary integral formulation of the velocity potential on the spheroid surface. Panel discretisation of the boundary integral is evaluated by the Hess–Smith [13,14] panel integral of the Rankine source potential and the panel integral of the harmonic function expansion. The proposed method is validated from the comparison of the proposed method’s results and earlier numerical results.

## 2. Initial formulation

Let  $L$  be the length of a spheroid submerged in a uniform stream of speed  $U$  in a fluid of infinite depth. The dimensionless fluid motion model transformed by  $L$ , the typical length of the motion, is described by Fig. 1. The Green function  $G^\mu$  defined in (2) is non-dimensional so that the wave number  $\nu$  and the Froude number  $\operatorname{Fn}$  are

$$\nu = \frac{1}{\operatorname{Fn}^2}, \quad \operatorname{Fn} = \frac{U}{\sqrt{gL}} \tag{4}$$

for  $g$  the gravitational acceleration. For convenience, we use non-dimensional coordinate system  $(x, y, z)$  (see Fig. 1) formulated from the dimensional system  $(Lx, Ly, Lz)$ .

The total velocity potential scaled by  $L$  is described as

$$\Phi = U(x + \phi).$$

Here  $\phi$  denotes the perturbed velocity potential of the problem in dimensionless form and satisfies the Laplace equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad \text{for } z < 0, \tag{5}$$

the linear free surface boundary condition

$$\frac{\partial^2 \phi}{\partial x^2} + \nu \frac{\partial \phi}{\partial z} + \mu \frac{\partial \phi}{\partial x} = 0 \quad \text{for } z = 0 \tag{6}$$

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