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# A modified saddle point method for predicting sound fields above a non-locally reacting porous medium

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## HIGHLIGHTS

- Accurate asymptotic solution for predicting sound fields above a rigid porous medium.
- Development of modified saddle point analysis.
- Asymptotic analysis for examining the physical phenomenon of wave motion.
- Acoustic characterization of sound absorption materials.

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## ABSTRACT

This paper examines an asymptotic analysis for predicting sound fields above a rigid-frame porous medium, the so-called non-locally reacting porous medium. Their solutions can be represented by a direct wave term, a reflected wave term and a diffraction wave term. Exact and analytical solutions are available for the direct wave and the reflected wave from a perfectly hard ground. In the contrary, the diffraction wave term can only be cast in an integral form that is amenable to approximate analysis. A modified saddle-point method is explored to evaluate the diffraction integral asymptotically. Three different types of non-locally reacting surfaces, which are an extended reaction, a hard-backed layer, and an impedance-backed layer, were considered. The sound fields above these porous surfaces have the same form but they are different by an augmented diffraction term in the solutions. The analytical formula for the total sound fields, which can be stated in a closed form, offer a physically interpretable solution comprising of a direct wave and ground reflected wave terms. This latter term can further be split into a specularly reflected plane wave and ground wave components. A series of numerical comparisons have been conducted to validate the asymptotic analyses presented in this study. It has been demonstrated that the overall sound fields can be predicted well by the formula for all incidence angles and for a wide range of non-locally reacting porous surfaces.



## 1. Introduction

The study of sound propagation above a non-locally reacting plane boundary has been conducted for many decades. An exact formulation [1], precise asymptotic formulas [2,3] and accurate numerical solutions were developed for the porous half-space. A more difficult problem involves the presence of a porous layer where the porous frame is motionless. The porous layer is often set on a rigid backing that has an application for acoustical characterization of the non-locally reacting sound absorbing materials from the measured sound fields [4–6]. This technique depends critically on the availability of an

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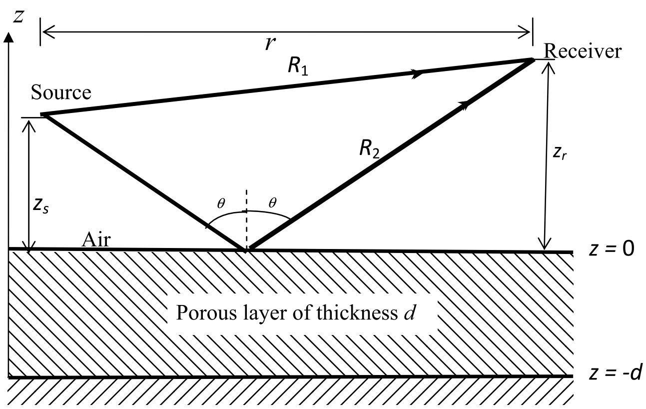


**Fig. 1.** An illustration of a monopole source and receiver placed above a layer of a porous medium.

accurate numerical solution that can be used in the inversion process. Allard et al. [7] derived an asymptotic formula for the sound field above a hard-backed porous layer. More recently, Li and Liu [8] extended their formula to include the prediction of sound fields above an impedance-backed porous layer.

In recent years, numerical intensive schemes (e.g. a finite element method [9] and boundary integral methods [10]) have been developed to calculate the sound fields above non-locally reacting surfaces. These numerical methods provide accurate solutions which are particularly suitable for analyzing low-frequency sound fields and for two-dimensional problems. It is because the number of grid points becomes prohibitive large for high source frequencies in a three-dimensional space. On the other hand, the analytical solutions according to the asymptotic formulas are well-known for their accuracies in predicting three-dimensional sound fields at high frequency or at long range [8,11]. The numerical solutions computed by the asymptotic formulas are often used to benchmark those calculated by the computationally intensive methods. Therefore, there is a need to provide accurate and versatile asymptotic solutions for predicting sound fields above a non-locally reacting medium.

Most of the earlier asymptotic analyses were based on a double saddle-point method [12] supplemented by the method of pole subtraction [13] for the sound fields above a locally reacting or a non-locally reacting plane boundary. This asymptotic solution is apparently singular when the source lies directly above (or below) the receiver or when the specific acoustic impedance of the boundary is equal to 1. Because of this singularity, the derived asymptotic formula cannot be used conveniently in a more general application. Hence, one of the motivations of the present study is to develop a more robust asymptotic formula for predicting sound fields above a non-locally reacting porous medium.

There is a less popular asymptotic approach, known as a modified saddle-point method developed by, Orr [14], Pauli [15] and Clemmow [16]. This asymptotic method was subsequently used by Kawai et al. [17] who derived the monopole sound-field propagation above a locally reacting plane. Chandler-Wilde and Hothersall [18] generalized the procedure to give the solution for the cylindrical-wave reflection in the form of an asymptotic series with an accurate estimation of the error bound. In the present study, it is aimed to use the modified saddle-point method for developing an asymptotic series to predict the sound fields above three types of non-locally reacting porous media: half-space, hard-backed and impedance-backed rigid porous layers. The solution is expressed in a form comparable to the classical Weyl–van der Pol formula [11,13] which offers a physical understanding of the problem. In addition, this paper and a related study [19] report an endeavor to apply the modified saddle-point method for studying the propagation of sound in the vicinity of the non-locally reacting porous medium.

To obtain numerical solutions, the first two terms of the asymptotic series based on the modified saddle-point method are used to compute the sound fields. The numerical results according to the asymptotic solution are validated by comparing with those obtained by other computationally intensive, but more accurate, schemes.

## 2. An integral formulation

Fig. 1 shows the geometrical configuration of the problem for determining the sound fields due to a monopole source above a flat porous medium with a layered structure. The time dependent factor, $e^{-i\omega t}$, is assumed and suppressed throughout the analysis. The wavenumber $k$, the density $\rho$, and, the speed of sound $c$ in the upper medium will be used in the following analysis. An axi-symmetric solution is sought where the sound fields can be conveniently split into three terms as follows [8]:

$$p = \frac{e^{ikR_1}}{4\pi R_1} + \frac{e^{ikR_2}}{4\pi R_2} + p_D. \tag{1}$$

The first and second terms of Eq. (1) are the sound fields due to the source and its image above a rigid ground with the respective distances of $R_1$ and $R_2$. The third term $p_D$ is the diffraction of sound by the flat porous surface. It is dependent on