Wave Motion 51 (2014) 240–255

Contents lists available at ScienceDirect

Wave Motion

journal homepage: www.elsevier.com/locate/wavemoti

Generation of stable solitary waves by a piston-type wave maker

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HIGHLIGHTS

- A numerical model for solitary wave generated by using a piston-type wave maker is developed.
- The method used in this numerical model is a meshless one.
- We study on how to generate "stable" solitary waves by using this numerical model.

ARTICLE INFO

Article history:
Received 6 January 2013
Received in revised form 24 July 2013
Accepted 25 July 2013
Available online 9 August 2013

Keywords:
Solitary wave
Wave generation
Piston-type wave maker
Meshless numerical model

ABSTRACT

The focus of present study is on how to generate solitary waves as pure as possible by using a piston type wave maker. A meshless numerical model, which can simulate the trajectories of fluid particles in a wave motion exerted by the wave paddle, is established for the purpose of present study. The present numerical model is verified by the comparison with experimental data before it is employed to the focused problem. Various wave paddle motions are considered. The results show that solitary waves generated by applying Fenton's solitary solution to the paddle motion proposed by Goring are purer than those generated by other paddle motions.

1. Introduction

The concern of this study is on how to use a piston type wave maker to generate solitary waves as "pure" as possible. A "pure" solitary wave means it has stable amplitude and minimized trailing waves during propagation. The pureness of solitary waves is significant in the study of solitary wave reflection [1] or interaction of solitary waves with either solitary waves or monochromatic waves [2].

The discovery of solitary waves is a remarkable scientific achievement [3]. Further experimental and theoretical study [4] showed that any net positive volume of water above the still water level leads to the eventual emergence of at least one solitary wave followed by a train of dispersive waves. Since then, various procedures for generating solitary waves have been studied. Methods for laboratory generation of solitary waves include dropping weights [3,5], displacing a given mass of water by a rising bottom [6], releasing a prescribed amount of water behind a barrier [7], and horizontal movement of a vertical paddle by a piston-type wave maker [8,9]. Among these methods, solitary wave generation using piston-type wave makers has been the most commonly employed method.

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0165-2125/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.wavemoti.2013.07.005

By assuming the wave to be of permanent form during the generation process, Goring derived a formula to determine the wave paddle trajectory [9] during the solitary wave generating procedure. It is assumed that the average horizontal water particle velocity adjacent to the wave paddle, $\bar{u}$, equals the wave paddle velocity

$$\frac{d\xi}{dt} = \bar{u}(\xi, t) = \frac{C\eta|_{x=\xi}}{h + \eta|_{x=\xi}} \tag{1}$$

where $\xi(t)$ is the position of the wave paddle at time t, while t is elapsed time since the start of the motion, C is the wave celerity, η is the free surface displacement, and h is the still water depth. In Ref. [9] it was proposed to use the solitary wave solution of Boussinesq [10] to determine the free surface displacement η and the wave celerity C for Eq. (1). This method is named as Goring's methodology for solitary wave generation in this paper. This has been the primary method of solitary wave generation for recent decades.

In the numerical study of Ref. [11], it could be found that Goring's methodology for solitary wave generation only works accurately for small waves. For a higher solitary wave, a slight depression of the free surface was observed behind a generated solitary wave accompanying with the wave height decrease when the wave propagates. The viscous effect was ignored in that numerical model so the fluid viscosity does nothing to the decay of wave height. Because the focus of that paper was on solitary wave run-up onto steep slopes, rather than on how to generate solitary waves, this phenomenon was just observed, but not further discussed. The depression behind the generated solitary wave could also be found in the figures of Refs. [12,13].

Experiments illustrated in Ref. [14] showed that the wave paddle motion derived by using the solitary wave solution of Rayleigh [15] in Eq. (1) could produce "purer" and more rapidly established solitary waves. However, because the boundary outskirt decay coefficient for shaping the solitary wave is smaller and thus more volume of water is pushed to form the wave, the produced waves are higher than the aimed waves. For this sake, the height of a generated solitary wave is hard to predict precisely when the stroke of the piston motion is given.

In Ref. [16], experimental results showed that even by applying Rayleigh's solitary wave solution [15] to Eq. (1), generated waves were all smaller than the aimed waves. For this sake more water needs to be pushed by the wave paddle so that waves generated could be higher and closer to the aimed waves. A new methodology was thus proposed in Ref. [16] by considering the evolving nature of the wave during the generation process. However, that new methodology conflicts with what was observed in Ref. [14], whose conclusion implies that Goring's wave paddle motion formula (Eq. (1)) is satisfactory but in the formula the solution to describe the wave profile should be reconsidered.

Inspired by Ref. [14], various solitary wave solutions are tested in this paper by carrying out numerical simulations. For this purpose, a numerical model is developed to describe the movement of fluid particles in the wave motion exerted by the wave paddle. Four solitary wave solutions for determining the free surface displacement η and the wave celerity C in Eq. (1) are considered. They are the solution of Boussinesq [10], the solution of Rayleigh [15], the solution of Grimshaw [17] and the solution of Fenton [18]. Before applying to the study on the accuracy of generated solitary waves, the numerical model is verified by the comparison with experimental data in Ref. [13].

2. Mathematical description of the free-surface wave problem

For several decades, water-wave problems have been studied as potential-flow problems governed by the Laplace equation with nonlinear free surface boundary conditions. Most of these studies have been carried out with boundary-element methods (BEMs), subjected to a mixed Eulerian–Lagrangian (MEL) time marching approach [19–22]. In present study, the assumption of potential-flow is followed. The governing equation is the Laplace equation

$$\nabla^2\phi = 0 \tag{2}$$

where $\phi(\overrightarrow{x}, t)$ is the velocity potential with $\overrightarrow{x}$ being the position vector and t being the time. The velocity vector is defined as $\overrightarrow{v} = \nabla\phi$. The kinematic free-surface boundary condition (KFSBC) and dynamic free-surface boundary condition (DFSBC) can be expressed as

$$\nabla\phi = \frac{d\overrightarrow{x}}{dt} \tag{3}$$

and

$$\left.\frac{\partial\phi}{\partial t}\right|_{z=\eta} = -g\eta - \frac{1}{2}(\nabla\phi\cdot\nabla\phi)_{z=\eta} + C(t) \tag{4}$$

where g is the gravitational acceleration, z is the vertical coordinate, $C(t)$ is Bernoulli's constant, which can be set to zero for a quiescent ambiance, and $\eta(x, y, t)$ is the free-surface displacement. The KFSBC is stated in the Lagrangian frame, whereas the DFSBC is in the Eulerian frame. To be consistent with later numerical treatment, the DFSBC is converted to the Lagrangian frame as

$$\left.\frac{d\phi}{dt}\right|_{z=\eta} = -gz + \frac{1}{2}\nabla\phi\cdot\nabla\phi. \tag{5}$$