Wave Motion 51 (2014) 308–322

Contents lists available at ScienceDirect

Wave Motion

journal homepage: www.elsevier.com/locate/wavemoti

In-plane wave motion and resonance phenomena in periodically layered composites with a crack

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HIGHLIGHTS

- Wave propagation in periodically layered elastic composites with a crack is studied.
- Dependence on crack location, size and the angle of wave incidence is analyzed.
- Special attention is devoted to resonances and wave localization phenomena.
- Power-density and energy streamlines are important to understand phenomena.

ARTICLE INFO

Article history:
Received 21 December 2011
Received in revised form 7 June 2013
Accepted 23 August 2013
Available online 4 September 2013

Keywords:
Crack
Localization
Resonance
Boundary integral equation method
Band-gaps
Phononic crystals

ABSTRACT

This paper investigates the transmission and propagation of two-dimensional (2D) time-harmonic plane waves in periodically multilayered elastic composites with a strip-like crack. The total wave field in the composite structure is represented as a sum of the incident wave field determined by the transfer matrix method and the scattered wave field described by integral representations in terms of the Green's matrices and the crack-opening-displacements. A numerical scheme is developed to compute the wave propagation characteristics and the crack-characterizing quantities. The effects of the crack location and size as well as the angle of wave incidence are investigated using the averaged crack-opening-displacements and the stress intensity factors. Special attention of the paper is devoted to resonance wave motion and wave localization phenomena in a stack of periodical elastic layers weakened by a single strip-like crack. Numerical results are presented and discussed to reveal the usual and the resonant wave transmission by using the power-density vector and the energy streamlines in the vicinity of the crack. Wave localization due to interior and interface cracks is analyzed by considering the energy captured by a crack, and resonance induced crack growth is also discussed.

1. Introduction

Periodic elastic composites also called phononic crystals have attracted an increasing research interest in recent years due to their unique dynamic properties as demonstrated by many researchers. They have potential innovative applications as multi-functional composites in acoustic and ultrasonic devices, wave filters, wave guides, wave splitter etc. [1,2].

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http://dx.doi.org/10.1016/j.wavemoti.2013.08.006

Wave propagation in periodic composite structures is often accompanied by band-gaps and localization phenomena, which are observed in photonic and phononic crystals [3]. Elastic composites are often also susceptible to damages during the manufacturing process as well as in service. In particular, delaminations at the imperfect interfaces between the constituents and also interior cracks within the individual layers can occur. Such interfacial or interior damages could change the dynamic properties of the periodic composites significantly and consequently cause noticeable alterations in band-gaps, wave transmission, wave localization etc. In general, a single defect like a crack changes wave pattern only locally in its surroundings, hence the induced wave localization and resonance may even lead to the final failure of the composite structure.

The scattering and diffraction analysis of elastic waves by defects is of special importance to non-destructive evaluation and characterization of materials and structures by using ultrasonic techniques. An overview on the mathematical models and numerical approaches related to crack analysis can be found in the review article [4] and the monograph [5]. A single crack in an unbounded, two-dimensional (2D) and linear elastic material has been investigated by Hijden and Neerhoff [6]. Layered half-spaces have been treated by Yang and Bogy [7]. Itou [8] and Qu [9] have considered an interface crack between two half-spaces, where the Green's function approach combined with different discretization schemes has been employed.

In the present paper, the propagation and transmission of 2D time-harmonic plane waves in multilayered elastic composites consisting of a stack of periodic elastic layers with a single crack is investigated. This study is a natural continuation of the author's previous works [10,11], where the anti-plane SH-wave propagation and transmission in a periodic array of elastic layers between two identical elastic half-planes with either a single crack or a periodic distribution of cracks has been analyzed. Unlike our previous investigations [10,11], this paper considers in-plane wave motion in periodically layered composites with a single crack. Accordingly, the present study is much more complex than that in [10,11]. For instance, in this paper we have to solve a system of integral equations for an unknown vector, while a single integral equation for a scalar unknown function has been solved in [10,11]. The total elastic wave field is represented as a sum of the incident wave field described by the transfer matrix method and the scattered wave field constructed via an integral representation [12]. The latter is based on an integral representation in terms of the Fourier-transform of the convolution of the Green's matrices and the crack-opening-displacements (CODs). The solution of the boundary integral equations arising from the stress-free boundary conditions on the crack-faces is obtained numerically by employing a Galerkin method with an expansion of the CODs into Chebyshev polynomials.

The developed numerical method presented in this paper is verified by using available reference solutions in literature for some special cases. Numerical results are given and discussed to reveal the essential phenomena of the elastic wave propagation and transmission in periodically multilayered elastic composite structures with a strip-like crack. Time-harmonic plane longitudinal (P-) and vertically polarized transverse (SV-) elastic waves with either normal incidence or oblique incidence are dealt with. Special focus of the paper is on the non-resonant and resonant wave of motion and wave localization near the crack. The effects of the crack location and size as well as the incidence angle are investigated in details. Standard and resonant wave propagation and transmission are illustrated via the power density vector and the energy streamlines in the vicinity of the crack, which provides us a deeper insight into the physics of the wave phenomena [13,14]. The energy amounts captured by an interior and an interface crack are compared, and a possible resonance induced crack growth is also discussed.

2. Mathematical model and the numerical solution method

Let us consider two-dimensional (2D) plane time-harmonic longitudinal (P) and transverse shear (SV) waves propagating in a multilayered elastic structure composed of two half-planes with a stack of N layers containing a strip-like crack of length $2l$ as shown in Fig. 1. The Cartesian coordinate system $\mathbf{x} = (x, z)$ associated with the center of the crack is introduced. The x-axis and the crack-faces are assumed to be parallel to the interfaces. The jth layer occupying the domain $|x| \le \infty$, $a_{j-1} \le z \le a_j$ of the thickness $d_j = a_j - a_{j-1}$ has the Lame's constants λ_j and μ_j and the mass density ρ_j. The subscripts $j = 0$ and $j = N + 1$ correspond to the lower and upper half-planes. The crack is located within the Mth layer at the distance d from the interface $z = a_{M-1}$.

The displacement vector for the 2D in-plane wave motion in the elastic material has two non-zero components denoted as $\boldsymbol{u}^j = u_i^j = \{u_x^j, u_z^j\}$, where the superscript j denotes the jth layer or the half-planes ($j = 0, j = N+1$). The material properties are determined by the Lame's constants λ_j and μ_j and densities ρ_j. The displacement components in homogeneous, isotropic and linear elastic materials can be written in terms of the longitudinal and the transverse wave potentials φ and ψ as

$$u_x = \frac{\partial\varphi}{\partial x} + \frac{\partial\psi}{\partial z}, \qquad u_z = \frac{\partial\varphi}{\partial z} - \frac{\partial\psi}{\partial x}.$$

Accordingly, the governing equations for time-harmonic wave motion within the jth layer $|x| \le \infty$, $a_{j-1} \le z \le a_j$ can be decoupled into two wave equations in terms of the potentials $\varphi = a_{1j}$ and $\psi = a_{2j}$ for the longitudinal and the transverse waves correspondingly

$$\frac{\partial^2 a_{rj}}{\partial x^2} + \frac{\partial^2 a_{rj}}{\partial z^2} = \frac{\omega^2}{c_{rj}^2} a_{rj}, \quad r = 1, 2, \tag{1}$$