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Scattering of monochromatic elastic waves on a planar crack of arbitrary shape

S. Kanaun *

Instituto Mexicano del Petróleo, Lazario Cardenes 25, Mexico D.F, 52399, Mexico

HIGHLIGHTS

- Monochromatic elastic wave scattering on a planar crack of arbitrary shape is considered.
- A 2D-integral equation for the crack opening vector is used.
- Gaussian functions are applied for discretization of the integral equation.
- Fast Fourier transform is employed for the solution of the discretized problem.
- Numerical solutions are compared with the results of other methods.

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ABSTRACT

Scattering of monochromatic elastic waves on an isolated planar crack of arbitrary shape is considered. The 2D-integral equation for the crack opening vector is discretized by Gaussian approximating functions. For such functions, the elements of the matrix of the discretized problem have forms of standard one-dimensional integrals that can be tabulated. For regular grids of approximating nodes, the matrix of the discretized problem has the Toeplitz structure, and the corresponding matrix–vector products can be calculated by the fast Fourier transform technique. The latter strongly accelerates the process of iterative solution of the discretized problem. Examples of calculations of crack opening vectors, dynamic stress-intensity factors, and differential cross-sections of circular (penny-shaped) and non-circular cracks for various incident wave fields are presented. For a penny-shaped crack and longitudinal incident waves normal to the crack plane, an efficient semi-analytical method of the solution of the scattering problem is developed. The results of both methods are compared in a wide frequency region of the incident field.

1. Introduction

The problem of plane monochromatic wave scattering on a planar crack of arbitrary shape in a homogeneous elastic medium has important applications in seismology, nondestructive evaluations of defects in solids, in the analysis of fracture processes by dynamic loading, etc. This problem has been in the focus of interest of many researchers for several decades. Basically this problem is firstly reduced to 2D-integral equations for the crack opening vector (see, e.g., [1–4]), and then, the boundary element method is used for the numerical solution of these equations. In this method, the crack surface is divided into a set of small subregions (boundary elements), and the unknown function (crack opening vector) is approximated by standard functions (e.g., polynomial splines) inside each element. The coefficients of the approximation become principal unknowns of the problem. Using the method of moments or the collocation method the problem is reduced to a finite

* Tel.: +52 55 58645665; fax: +52 55 58645651.
E-mail address: kanaoun@itesm.mx.

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system of linear algebraic equations for these coefficients (discretized problem). The elements of the matrix of the discretized problem are integrals over the boundary elements. Some of these integrals are singular, and difficulties of their calculations depend on the complexities of the boundary element shapes and types of approximation functions. In fact, a great portion of computer time is spent on the calculations of elements of the non-sparse matrix of the discretized problem. If dynamic stress intensity factors (SIFs) are of interest, high accuracy of the solution near the crack edge is required, and the dimensions of the matrix of the discretized problem have to be large. For such matrices, only iterative methods are efficient, and time-consuming operation of the matrix–vector product should be performed at each step of the iteration process.

For cracks of specific geometries, other strategies of the numerical solution of the scattering problem were proposed in [5–7] (penny-shaped cracks), [8] (rectangular cracks).

In [9,10], an efficient method for the numerical solution of integral equations of electromagnetic scattering problems was outlined. This method can be also applied to the solution of other scattering problems of mathematical physics. Discretization of the integral equations of these problems was proposed to carry out with the help of a set of identical approximating functions centered at the nodes of regular grids. As a result, the matrix of the discretized problem has Toeplitz's properties, and the corresponding matrix–vector products can be calculated by the fast Fourier transform (FFT) technique. The latter accelerates essentially the process of iterative solution of the discretized problems. For 3D-electromagnetic and elastic wave scattering, this method was employed in [11,12], where Gaussian approximating functions were used for discretization of the corresponding integral equations. The theory of approximation by Gaussian and other similar functions was developed by Maz'ya and Schmidt in [13]. The Gaussian functions were used for the numerical solution of the crack problem of static elasticity in [14]. In the present work, this method is extended to the problem of elastic wave scattering on a planar crack of arbitrary shape. In application to the crack scattering problem, the method has the following advantages.

- Calculation of the elements of the matrix of the discretized problem is reduced to a small number of standard one-dimensional integrals that can be tabulated. Thus, integration is excluded in fact from the process of this matrix construction.
- For regular node grids, the matrix of the discretized problem has Toeplitz properties, and a column of such a matrix contains all elements of the latter. So, the volume of calculations and required memory are essentially reduced in comparison with conventional methods of the numerical solution of the problem.
- Application of the FFT algorithms for calculation of matrix–vector products essentially accelerates the process of iterative solution of the discretized problem. As a result, the numerical solution for grids with 10^5–10^6 nodes is not time consuming and can be carried out by modest personal computers and notebooks.
- For the mentioned number of nodes (approximating functions), there is no need to use special boundary elements or fine grids in the vicinity of the crack edge for calculation of dynamic stress intensity factors because the accuracy of SIF calculations is sufficient for practical applications.
- The method is mesh free in fact, and only coordinates of the nodes and the values of the incident field at the nodes are necessary initial information for performing the method.

The structure of the paper is as follows. In Section 2, the integral equation of the scattering problem for a planar crack is discussed. In Section 3, approximation by Gaussian functions is considered. In Section 4, the integral equation of the scattering problem is discretized by Gaussian approximating functions. The elements of the matrix of the discretized problem are obtained in the forms of standard absolutely converging one-dimensional integrals. In Section 5, scattering of longitudinal and transversal waves of various lengths on a circular (penny-shaped) crack is considered. For longitudinal incident waves orthogonal to the crack plane, an efficient semi-analytical method of the solution is proposed. Results of both methods are compared in a wide frequency region of the incident field. In Section 6, the algorithm of calculation of stress intensity factors in the framework of the method is described. Scattering on a non-circular crack is considered in Section 7. Far scattered fields and differential cross-sections of planar cracks are considered in Section 8. Specific features of the method and the area of its application are discussed in the conclusion.

2. Integral equations of the scattering problem for a homogeneous medium with an isolated planar crack

Consider an infinite homogeneous medium with the density ρ and elastic stiffness tensor C_{ijkl} that contains an isolated planar crack Ω, Γ is the crack boundary contour (Fig. 1). Let a monochromatic elastic wave of displacements $\mathbf{U}^0(x,t)$ (incident field)

$$\mathbf{U}^0(x,t) = \mathbf{u}^0(x)e^{i\omega t}, \qquad \mathbf{u}^0(x) = \mathbf{a}e^{-iq(\mathbf{n}^0\cdot\mathbf{x})} \tag{1}$$

propagate in the medium and be scattered on the crack. Here ω is frequency, t is time, q is the wave number of the incident field, $\mathbf{n}^0$ is its wave normal, and $\mathbf{n}^0\cdot\mathbf{x}$ is the scalar product of the vectors $\mathbf{n}^0$ and $\mathbf{x}$ ($\mathbf{x}$ is the vector of a point x in the 3D-medium); $\mathbf{a}$ is the incident wave polarization vector. For longitudinal incident waves in an isotropic medium,

$$q = \alpha = \sqrt{\frac{\rho}{\lambda+2\mu}}\,\omega. \tag{2}$$