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Anomalous propagation of acoustic traveling waves in thermoviscous fluids under the Rubin–Rosenau–Gottlieb theory of dispersive media

P.M. Jordan [a,*], R.S. Keiffer [a], G. Saccomandi [b]

[a] Acoustics Div., U.S. Naval Research Laboratory, Stennis Space Ctr., MS 39529, USA

[b] Dipartimento di Ingegneria Industriale, Università degli Studi di Perugia, 06125 Perugia, Italy

HIGHLIGHTS

- Nonlinear acoustic waves in thermoviscous fluids are examined under RRG theory.
- A nonlinear version of the van Wijngaarden–Eringin equation is derived.
- An exact traveling wave solution in the form of a kink is determined.
- The existence of an anomaly in the kink's wave speed is established.
- Possible explanations of the anomaly are noted.

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ABSTRACT

We report on an apparent anomaly associated with the compressible, thermoviscous version of what has come to be known as RRG theory. The difficulty in question, which does not appear to impact the lossless special case of this theory, manifests itself under the traveling wave assumption and has the shock-front of the resulting kink waveform and the (instigating) piston propagating in *opposite* directions.

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1. Introduction

In 1995, Rubin et al. [1] presented a phenomenological-based theory of generalized continua, which we shall refer to henceforth as RRG, the purpose of which is modeling dispersive effects caused by the introduction of a medium's characteristic length, which is usually denoted by α. According to RRG, α is regarded as an inherent material property; indeed, as Destrade and Saccomandi [2] were the first to show, RRG is a special case of the *theory of simple materials*, where "simple materials" refers to the general class of continua wherein the Cauchy stress is determined by the entire history of the deformation gradient. As such, RRG can readily be generalized to describe the mechanics of an extremely diverse range of continua; see, e.g., Refs. [2,3]. From the modeling point of view, the most appealing feature of RRG theory is the following: *Only* the Helmholtz free energy and the Cauchy stress are modified; however, these modifications are achieved by simply adding perturbative terms, which must satisfy certain constraint equations, to the constitutive relations of the former and latter.

* Corresponding author. Tel.: +1 228 688 4338.
E-mail address: pedro.jordan@nrlssc.navy.mil (P.M. Jordan).

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The present study has been motivated by those of Keiffer et al. [4], who investigated acoustic traveling waves under the simplest possible Lagrangian-averaged Euler-α (LAE-α) model of compressible flow, and Jordan and Saccomandi [5], who considered acoustic traveling waves in lossless fluids described by a nonlinear special case of RRG. Unlike these earlier studies, however, here, the effects of thermoviscous dissipation *are* taken into account; specifically, we examine acoustic traveling waves in the "RRG version" of a thermally conducting gas that obeys Newton's viscosity law. As we shall see, the introduction of the length scale α yields what can be described as a "nonlinearly dispersive" version of the Navier–Stokes equations, wherein *only* the momentum equation is modified via the introduction of terms with coefficient α.

The purpose of the present communication is threefold: (I) Derive the equations describing acoustic propagation in thermoviscous perfect gases under the RRG formulation; (II) determine and analyze the properties of the traveling wave solution (TWS) of the ensuing weakly nonlinear equation of motion; and (III), point out an apparent propagation anomaly exhibited by the aforementioned TWS.

2. Constitutive relations

We begin by listing the RRG-modified constitutive relations describing acoustic propagation in thermoviscous perfect gases, the transport coefficients of which we regard as constants, and within which the flow of heat is described by Fourier's law of heat conduction, namely,

$$\boldsymbol{q} = -K\nabla\vartheta, \tag{2.1}$$

where $\boldsymbol{q}$ denotes the heat flux vector, $\vartheta(>0)$ is the absolute temperature, and K represents the (constant) thermal conductivity.

Now, let $\boldsymbol{X}$ denote the position of a material point in the reference configuration and $\boldsymbol{x}$ denote the position of the same material point in the present configuration at time t. Let $\boldsymbol{F} = \partial\boldsymbol{x}/\partial\boldsymbol{X}$ be the deformation gradient and $\boldsymbol{L} = \dot{\boldsymbol{F}}\boldsymbol{F}^{-1}$ the velocity gradient. By $\boldsymbol{D}$, as usual, we denote the symmetric part of $\boldsymbol{L}$. Under RRG, the Cauchy stress tensor, $\boldsymbol{T}$, is expressed as the sum $\boldsymbol{T} = \boldsymbol{T}^{(1)} + \boldsymbol{T}^{(2)}$. In the present study, the first term of this sum is the well known constitutive relation for a compressible Newtonian fluid, namely,

$$\boldsymbol{T}^{(1)} = -\wp\boldsymbol{I} + 2\mu\boldsymbol{D} + \left(\mu_{\mathrm{B}} - \tfrac{2}{3}\mu\right)(\nabla\cdot\boldsymbol{u})\boldsymbol{I}. \tag{2.2}$$

Here, $\boldsymbol{u}$ is the velocity vector; $\wp$ is the thermodynamic pressure; $\mu(>0)$ and $\mu_{\mathrm{B}}(\geq 0)$ denote, respectively, the coefficients of shear and bulk viscosity; and $\boldsymbol{I}$ is the identity tensor. The second term, which is the non-standard part of $\boldsymbol{T}$, represents the perturbation induced by the introduction of material dispersion. Adopting the particular case of ψ_2 posited in Ref. [1, Eq. (20)], i.e.,

$$\psi_2 = \alpha^2\boldsymbol{D}^2, \tag{2.3}$$

where $\alpha(\geq 0)$ is a constant which carries the (SI) unit of metre, it follows that

$$\boldsymbol{T}^{(2)} = \varrho\alpha^2\left[(\partial\boldsymbol{a}/\partial\boldsymbol{x}) + (\partial\boldsymbol{a}/\partial\boldsymbol{x})^{\mathrm{T}} + 2\boldsymbol{L}^{\mathrm{T}}\boldsymbol{L} - 4\boldsymbol{D}^2\right]. \tag{2.4}$$

Here, $\boldsymbol{a}$ denotes the acceleration vector; $\varrho(>0)$ is the mass density; and ψ_2 represents the perturbation applied to ψ_1, the classical specific Helmholtz free energy [6].

As the reader has likely already inferred, under RRG the specific Helmholtz free energy becomes $\psi = \psi_1 + \psi_2$; and therefore,

$$\psi = (e_1 - \vartheta\eta) + \psi_2, \tag{2.5}$$

since $\psi_1 = e_1 - \vartheta\eta$. Here, e_1 represents the classical (i.e., unperturbed) specific internal energy and η denotes the specific entropy. In the case of a perfect gas, to which we have confined our focus, the thermodynamic variables satisfy the equation of state

$$\wp = (c_p - c_v)\varrho\vartheta, \tag{2.6}$$

also known as the *perfect gas law*, and

$$e_1 = c_v\vartheta, \tag{2.7}$$

where the constants $c_p > c_v > 0$ denote, respectively, the specific heats at constant pressure and volume [6]. It should be noted, however, that since the *Gibbs relation* [6, p. 58], which in the case of a perfect gas can be expressed as

$$\vartheta\,\mathrm{d}\eta = \mathrm{d}e_1 - c_v(\gamma - 1)\vartheta\varrho^{-1}\mathrm{d}\varrho, \tag{2.8}$$

is *not* modified under RRG, we can, in the present context, recast (2.6) as

$$\vartheta = \vartheta_{\mathrm{e}}(\varrho/\varrho_{\mathrm{e}})^{\gamma-1}\exp[(\eta - \eta_{\mathrm{e}})/c_v], \tag{2.9}$$

which, of course, is also true under classical gas dynamics theory [7, Eq. (1.1)]. Here, $\gamma = c_p/c_v$ denotes the adiabatic index and, henceforth, the equilibrium state values of the field variables, which are those appended with an "e" subscript, shall be regarded as constants.