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Unconventional wave reflection due to "resonant surface"

L. Schwan, C. Boutin *

Université de Lyon, Ecole Nationale des Travaux Publics de l'Etat, LGCB, UMR CNRS 5513, CeLyA, Rue Maurice Audin, 69518 Vaulx-en-Velin, France

HIGHLIGHTS

- A drastic change of *P* and *SV* waves conversion.
- The full depolarization of normally-incident shear waves.
- The conversion of *SH* waves into *P* and *SV* waves.
- The possibility of the whole reflected field to vanish.

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ABSTRACT

This study deals with the reflection phenomena in an elastic half-space on which lies a "resonant surface". The resonant surface consists in a 2D periodic repetition of a surface element over which linear oscillators are distributed. Following the homogenization approach developed by Boutin and Roussillon (2006) [1], the periodic distribution of oscillators (1 to 3D sprung-mass) is reduced to a frequency-dependent surface impedance. It is hereby shown that the surface motion comes to zero in the resonating direction around the oscillators' eigenfrequency. Further, the surface impedance may be isotropic or anisotropic, according to the type of oscillator. Thereby unusual free/rigid mixed boundary condition arises, which in turn induces atypical reflected wave fields. The most notable effects are (i) drastic change of *P* and *SV* waves conversion, (ii) depolarization of shear waves, (iii) conversion of *SH* waves into *P* and *SV* waves, and (iv) possibility of vanishment of the whole reflected field. The physical insight of the theoretical results is discussed and numerical illustrations are provided.

1. Introduction

The present study is concerned with the reflection phenomena in an elastic homogeneous half-space on which lies a "resonant surface". The resonant surface consists in linear oscillators distributed on the "free" surface with sufficient regularity to consider that the "oscillator layer" is characterized by a representative surface element (RSE) containing a few oscillators.

The simplest realization of this situation is a 2D periodic repetition of the same representative surface element Σ over which linear oscillators are distributed (Fig. 1). Such specific configurations are encountered in different domains of application and at different scales, according to the nature of the oscillators and of the supporting medium: for instance in geophysics, when considering the skyscraper-city effect on seismic motions e.g. [2–4]; in ultrasound survey, when inverting signals in the presence of highly corrugated surface; in dynamics of nanostructures, with nanotubes or nanocrystals equidistantly grown on the substrate [5].

* Corresponding author.
E-mail addresses: logan.schwan@entpe.fr (L. Schwan), claude.boutin@entpe.fr (C. Boutin).

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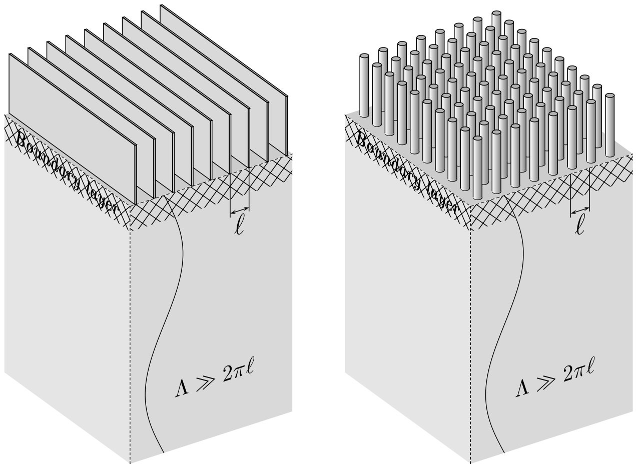

Fig. 1. Basic examples of resonating surface.

We address long wave propagation in the sense that the wave-lengths in the media are much larger than the size of the period (or of the RSE). In other words, a scale-separation condition is satisfied. The key point of the study is to assume that the resonance of the oscillators occurs within the scale-separation frequency range.

In this framework, the paper [1] demonstrates theoretically through multiple scale homogenization method [6,7] that the resonant surface can be described at the leading order by an equivalent boundary condition. This latter is formulated as a macroscopic impedance condition which frequency-dependent expression is directly related to the features of the oscillator.

The aim of the present paper is to systematically investigate the unusual effects related to the resonance of the oscillators and to evidence how much the reflected field differs from the field reflected with a free surface or with a rigid upper-layer configuration. For simplicity, we consider a single oscillator on the period, with three degrees of freedom associated to the horizontal or vertical directions. Isotropic (resp. anisotropic) horizontally resonant surface is obtained when the oscillator presents the same (resp. different) features in both horizontal directions. This simple situation underlines the key aspects of the phenomena, that can be extended with similar principles to more complex cases.

The paper is organized as follows. In Section 2, the basic assumptions and essential aspects of the modeling of a resonant surface by an equivalent impedance are briefly recalled. In Section 3, a general formulation of the wave reflection in presence of a surface impedance is proposed. Section 4 is devoted to the effect of isotropic horizontally resonant surfaces on the reflection of incident *SH*, *SV* and *P* plane waves. Sections 5 and 6 deal with the same questions for vertically resonant surfaces and anisotropic horizontally resonant surfaces.

2. Equivalent impedance of a "resonant surface"

This section sums up the principles governing the theoretical derivation of the equivalent impedance of a resonant surface (for more details see [1]).

2.1. Statement of the boundary layer problem

Consider a Σ-periodic distribution of linear oscillators lying on top plane surface Γ (of outward normal $\mathbf{n}$) of a homogeneous isotropic half-space of elastic tensor $\mathbf{C}$ (Lamé coefficients λ, μ) and density ρ. In this linear system, we study the propagation of harmonic waves of frequency $f = \omega/2\pi$, assuming a scale separation between the characteristic size ℓ of Σ ($\ell = O(\sqrt{|\Sigma|})$) and the shear wavelength Λ in the medium, thus:

$$\varepsilon = 2\pi\ell/\Lambda \ll 1 \quad \text{where } \Lambda = \frac{2\pi c_S}{\omega},\ c_S = \sqrt{\frac{\mu}{\rho}}. \tag{1}$$

The oscillators set in motion by the waves induce on Γ a heterogeneous distribution of stresses, $\sigma.\mathbf{n} = \mathbf{t}\exp[-i\omega t]$ (in the sequel the term $\exp[-i\omega t]$ is systematically omitted). Because of periodicity and scale separation, (i) at the micro-scale, $\mathbf{t}$ is locally Σ-periodic, (ii) the distribution $\mathbf{t}$ also varies at the wavelength scale. The local 2D periodicity of $\mathbf{t}$ enforces the same 2D periodicity of the micro-scale perturbations of the physical quantities in the medium. Furthermore, as the sources of micro-variations are located on Γ, it is inferred that away from Γ small scale variations vanish, while macro-variations remain. Such a situation corresponds to a boundary layer located in the vicinity of the surface (see also in other contexts [6,8]).