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Parametric interaction of acoustic waves in micro-inhomogeneous media with hysteretic nonlinearity and relaxation

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HIGHLIGHTS

- An interaction of acoustic waves in hysteretic media with relaxation is studied.
- The case of the degenerate interaction is considered.
- The parameters of the weak and the powerful wave are determined.

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ABSTRACT

A theoretical investigation of parametric processes that arise as a result of the interaction of powerful and weak longitudinal acoustic waves in micro-inhomogeneous media with hysteretic nonlinearity and relaxation was carried out. The case of degenerate interaction between a powerful high-frequency wave and a weak low-frequency one was considered. The nonlinear damping coefficient and the carrier frequency phase delay of the weak wave propagating under the action of the powerful wave were determined.

1. Introduction

Propagation and interaction of acoustic waves in different media are accompanied by various nonlinear phenomena: generation of higher harmonics and combination frequencies; self-action, modulation, and demodulation of the waves; sound-by-sound attenuation, etc. The amplitude–frequency dependences of these phenomena are determined by the nonlinear properties of the medium. Thus, they can be used to study the medium's acoustic nonlinearity mechanisms and to develop sensitive methods of nonlinear diagnosis. A theory of nonlinear wave processes (NWPs) in weakly nonlinear homogeneous solids is currently complete; it is based on "classical" five-constant elasticity theory [1–3]. From it follows that a state equation of media for longitudinal stresses σ and strains ε is characterized by elastic quadratic nonlinearity with no inertia properties. In this case, longitudinal waves synchronously interact only at their collinear propagation because of the absence of the sound velocity dispersion [1–4].

Together with homogeneous media, a wide class of so-called micro-inhomogeneous media exists in nature. Micro-inhomogeneous [4,5] (or mesoscopic [6,7]) media contain defects that are greater in size than the interatomic distances but less than the acoustic wavelength. In these media there are many defects per wavelength and their spatial distribution is statistically homogeneous. Thus, on average, it is possible to consider them to be "acoustically homogeneous" or "macro-homogeneous" on a scale larger than the size of defects but smaller than the wavelength. These media are characterized by

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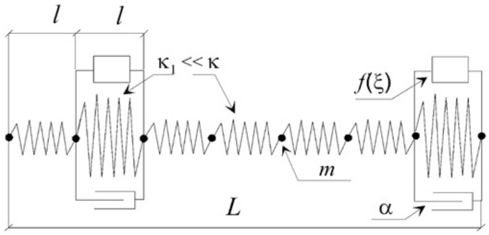

Fig. 1. Rheological model of nonlinear micro-inhomogeneous media with relaxation.

an anomalously high and (generally speaking) frequency-dependent "nonclassical" acoustic (often hysteretic) nonlinearity, which is due to the defects present in their structure, such as cracks, grain boundaries, dislocations, etc. The relatively high compressibility and concentration [8–12] of these nonlinear defects determine the high acoustic nonlinearity of similar media (in comparison with homogeneous ones). No universal nonlinear equation of state has been derived for micro-inhomogeneous media, and, therefore, development of the theory of NWPs is rather topical for modern nonlinear acoustics. The fundamental purpose of such research is to elucidate the physical mechanisms of anomalous nonlinearity and to derive a nonlinear dynamic equation of state, just as it was possible to develop sensitive nonlinear diagnostic techniques for micro-inhomogeneous media. Much research has been devoted to this subject [7,13,8–12,14,15]. However, many related problems remain unsolved because of their complexity. In particular, wave interactions in media with hysteretic nonlinearity are difficult to describe analytically because of the non-analytic nature of the nonlinearity in their equations of state and the specific nonlinear "memory" of the preceding locally maximal (in magnitude) strain value in the medium, i.e., the wave amplitude. Generally speaking, real micro-inhomogeneous media also possess an ordinary linear relaxation, which leads to the frequency dependence (i.e. dispersion) of their linear and nonlinear acoustic properties. This fact complicates the solution of wave interaction problems in such media even more.

Experimental and theoretical studies testify that many micro-inhomogeneous media (in particular, metals and rocks) possess hysteretic nonlinearity. According to the generally accepted point of view [16–19], the manifestations of hysteretic nonlinearity in polycrystalline solids are due to dislocations, i.e., one-dimensional defects of the crystal lattice. The "spectrum" of possible nonlinear phenomena (NPs) in hysteretic media is richer than in conventional homogeneous media, and the regularities of NP manifestations in micro-inhomogeneous hysteretic media are qualitatively different from those in homogeneous media with an analytic power-law quadratic nonlinearity. For example, in media with quadratic elastic nonlinearity (at small distances from the source, long before the appearance of ambiguity, i.e., tipping over, in the wave profile), the nth harmonic amplitude is proportional to the nth power of the primary wave amplitude propagating with a constant velocity without damping [2,3]. However, in media with quadratic hysteretic nonlinearity, amplitude-dependent internal friction (ADIF) phenomena including nonlinear loss and defect of elastic modulus are observed. They are proportional to the primary wave amplitude. In addition, higher harmonic generation takes place, with harmonic amplitudes being proportional to the square of the primary wave amplitude [8,11]. Analogous distinctions between "analytical" and hysteretic media will be exhibited at the parametric interaction of acoustic waves with different amplitudes and frequencies. In particular, a powerful pump wave will modulate and change the parameters of hysteretic media and, by doing so, influence the conditions of weak (test) wave propagation.

The present work assumes the results of theoretical studies (by the perturbation method) of parametric phenomena arising from the propagation and interaction of low-frequency (LF) and high-frequency (HF) longitudinal acoustic waves in micro-inhomogeneous media with both hysteretic nonlinearity and relaxation.

2. Equation of state of the micro-inhomogeneous medium with hysteretic nonlinearity and relaxation

A rheological model of nonlinear micro-inhomogeneous media with relaxation was proposed in [20] (Fig. 1). This model is in the form of an inhomogeneous chain consisting of sequential point masses m as well as perfectly elastic linear elements and relatively scarce nonlinear viscoelastic elements (including hysteretic ones) connected in series. The chain's homogeneous parts correspond to regions of defect-free perfectly elastic solid. They consist of point masses and rigid elastic links-springs characterized by the coefficient of elasticity κ and the length l. The other parts consisting of nonlinear viscoelastic elements (characterized by the relative coefficient of elasticity $\zeta = \kappa_1/\kappa < 1$ and the same length l) correspond to defects. Let us consider that this number of all elements on length $L \ll \Lambda$ (Λ is the length of the wave) is equal to N ($L = Nl \gg l$) and that the number of defects $N_d = \nu N$. So their relative concentration is equal to $\nu = N_d/N$. In low-frequency range, when $\omega \ll \sqrt{\kappa/m}$, the state equation of each from these defects (i.e., the strain ξ dependence of the stress σ) is defined by the following expression:

$$\sigma(\xi,\dot{\xi}) = \zeta E[\xi - f(\xi)] + \alpha\dot{\xi}, \tag{1}$$

where $E = \kappa l$ is an elastic modulus of rigid elements, $f(\xi)$ is the strain nonlinear function including the hysteretic one ($|f(\xi)| \ll |\xi|$), $\dot{\xi}$ is the strain rate, ω is the frequency of deformation, and α is the viscosity coefficient.